Solution to pract. final:

8) i) $y' = 3x + 2y$, $y(0) = 1$

$h = 0.2$ on $0 \leq x \leq 1$. Euler's method

$x_0 = 0$

$y_0 = 1$

$x_1 = x_0 + h = 0.2$

$y_1 = y_0 + h\, f(x_0, y_0)$

$= 1 + 0.2 \times 2$

$y_1 = 1.4$

$x_2 = x_1 + h = 0.4$

$y_2 = y_1 + h\, f(x_1, y_1)$

$= 1.4 + 0.2\, f(0.2, 1.4)$

$y_2 = 2.08$

And we keep going. We get the following table:

n	0	1	2	3	4	5
x_n	0	0.2	0.4	0.6	0.8	1
y_n	1	1.4	2.08	3.152	4.7728	7.16192

Solution to pract. final: 2/2

8) (continued) Improved Euler

$y' = 3x + 2y$, $y(0) = 1$

$h = 0.2$, $0 \le x \le 1$

$x_0 = 0$

$y_0 = 1$

$k_1 = f(0, 1) = 2$

$u_1 = y_0 + h k_1 = 1.4$

$k_2 = f(x_1, u_1) = f(0.2, 1.4) = 3.4$

$y_1 = y_0 + h \frac{1}{2}(k_1 + k_2) = 1.54$

And we keep going. We get the following table:

n	0	1	2	3	4	5
x_n	0	0.2	0.4	0.6	0.8	1
k_1		2	3.68	6.166	9.846	15.292
u_n		1.4	2.276	3.716	5.992	9.504
k_2		3.4	5.752	9.232	14.384	22.008
y_n	1	1.54	2.483	4.023	6.446	10.176

provided I made no calculation mistakes.

ii) is similar..