

Solution to practice final:

1/1

10) solve using the method of elimination:

$$\frac{dx}{dt} = Ax \quad , \quad A = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}$$

$$\begin{cases} x' = x - y \\ y' = x + 3y \end{cases} \quad (*)$$

Let  $D = \frac{d}{dt}$ . Then we can rewrite \* as:

$$\begin{pmatrix} D-1 & 1 \\ -1 & D-3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

If we apply  $D-1$  to the second row and add the 1 row, we get:

$$((D-1)(D-3) + 1)y = 0$$

$$\Rightarrow (D^2 - 4D + 4)y = 0$$

$$\Rightarrow (D-2)^2 = 0$$

$$\Rightarrow \boxed{y = Ae^{2t} + Bte^{2t}}$$

From the 2<sup>nd</sup> equation of \*, we get:

$$x = y' - 3y$$

$$\Rightarrow \boxed{x = (B-A)e^{2t} - Bte^{2t}} \quad (\text{after simplifying})$$