MAT203 Spring 2010

Practice Midterm I

The actual midterm will contain six problems. You will not be allowed to use notes or calculators.

Problem 1 Which points are not collinear

1. $P(1,-1,3)$, $Q(7,5,3)$, $S(-11,-13,3)$, $T(19,17,3)$.
2. $A(1,2,3)$, $B(1,-2,1)$, $C(2,2,3)$, $D(3,4,2)$.

Problem 2 For vectors $v=(2,-2,0)$, $u=(1,1,1)$, $w=(1,-1,1)$ compute the quantities that make sense:

1. $v\cdot(u+w)$,
2. $v\cdot u+w$,
3. $v\times u+w$,
4. $(v\times u)\cdot w-u\cdot w$

Problem 3 Compute cosine of the angle between vectors. Determine which of the pairs are orthogonal, for which pairs the angle is acute, for which obtuse.

1. $v_1=(1,1,-3)$, $u_1=(-2,-2,6)$.
2. $v_2=(2,1,-2)$, $u_2=(-2,0,6)$.
3. $v_3=(3,3,-2)$, $u_3=(-1,-1,-3)$.
4. $v_4=(1,-2,3)$, $u_4=(3,2,1)$.

Problem 4 Find orthogonal projection of vector $v=(1,-1,1)$ onto vector $u=(1,5,2)$. Also find a component of v orthogonal to u.

Problem 5 Determine whether the set of four points belongs to a plane:

1. $P(0,1,2)$, $Q(-3,2,1)$, $R(2,-2,1)$, $S(-1,-1,0)$

2. $A(1,0,2), B(-2,3,2), C(0,-1,1), D(-1,1,0)$

Problem 6 Find equation of a plane that contains points

1. $P(1,-1,1), Q(-2,0,1), R(1,1,-1)$.
2. $A(-2,3,2), B(0,-1,1), C(-1,1,0)$
3. Compute the cosine of angle between these two planes.
4. Denote the plane that contains $A(-2,3,2), B(0,-1,1), C(-1,1,0)$ by K. Find the distance between $Q(-2,0,1)$ and K.
5. Sketch K.

Problem 7 Find parametric and symmetric equations of the line that passes through two points $P(1,-1,0)$ and $Q(-1,2,-3)$. What is the distance from the point $S(4,4,4)$ to this line?

Problem 8 Determine which of the following equations define a cylinder and sketch its graph:

1. $x^2 + y^2 = 1 + z^2$.
2. $\ln(x) = y$.
3. $\cos(z) = y$.

Problem 9 Classify the surface defined by the following equations

1. $x^2 - 2x - 4 - y^2 - 4y - z = 0$
2. $3x^2 - 6x + 8 + 2y^2 + 8y - z^2 + 2z = 0$
3. $x^2 + 2x + 1 - 2y^2 + 4y - z^2 + 2z = 0$

Problem 10 Find equation of a surface of revolution obtained by rotation the curve given by equation $y = \ln(x)$ about

1. x-axis.

2. y-axis.

Problem 11 1. A surface in rectangular coordinates is defined by equation

$$x^2 - y^2 = 1.$$

Find its equation in cylindrical and spherical coordinates.

2. A surface in spherical coordinates is given by

$$\rho \sin\phi \cos\theta + \rho \sin\phi \sin\theta - \rho \cos\phi = 1.$$

Find its equation in rectangular and cylindrical coordinates.