MAT203 Spring 2010

Practice Final

The actual Final exam will consist of twelve problems that cover sections 11.1-14.5 (inclusive)

Problem 1 Find a unit vectors that perpendicular to the graph of functions

1. $\sin(x^2)$ at point with x-coordinate $\sqrt{\frac{\pi}{3}}$

2. $\ln(1 + \cos(x))$ at point with x-coordinate $\frac{\pi}{6}$

Present two solutions. One should use the fact that a vector orthogonal to vector (a, b) has coordinates $(-b, a)$. The other solution should use gradients.

Problem 2 Give an example of three distinct points collinear to $P = (1, 2, 3)$ and $Q = (3, 2, 1)$.

Problem 3 Find orthogonal projection of vector $v = (1, -1)$ on a line l that contains vector $(1, 2)$. Also find the normal to l component of v.

Problem 4 Find the area of a parallelogram $ABCD$. The point A, B, C have coordinates $(1, 1, 1)$, $(1, -2, 3)$, $(2, -1, -1)$.

Problem 5 Determine the $\cos(\theta)$, where θ is the angle enclosed by two intersecting surfaces $x^2 - y^2 + z^2 = 1, x^2 + y^2 + z^2 = 3$ at a point $(1, 1, 1)$.

Problem 6 Determine the distance between point $(1, 0, 1)$ and a tangent plane to the surface

$$xyz = 1$$

at a point $(1, 1, 1)$.

Problem 7 Determine the distance between point $(1, 0, 1)$ and a tangent line to a curve given by parametric equation

$$x(t) = \sin(t)$$

$$y(t) = \cos(2t)$$

$$z(t) = \sin(3t)$$

at a point $t = \pi$.

Problem 8 Determine the type of the quadratic surface and draw the traces at $z = 1$, $y = 0$.

1. $x^2 + 2y^2 - z^2 = 3$

2. $x^2 - 2y^2 - z + 2x = 3$

3. $x^2 - 2y^2 + z^2 = -3$

Problem 9 Write equation of the surface $xyz = 1$ in

1. spherical

2. cylindrical

coordinates

Problem 10 Find

1. the init tangent vector

2. the principal unit normal vector

for the function $r(t) = t\mathbf{i} + t^2\mathbf{j} + \frac{t^2}{2}\mathbf{k}$ at $t = 1$

Problem 11 Find magnitudes of

1. tangential $a_{\mathbf{T}}$
2. notmal $a_{\mathbf{N}}$

components of acceleration of the function $r(t) = e^t\mathbf{i} + 2t\mathbf{j} + e^{-t}\mathbf{k}$ at $t = 1$

Problem 12 Find the curvature of the curve $r(t) = e^t\mathbf{i} + 2t\mathbf{j} + e^{-t}\mathbf{k}$ as a function of t

Problem 13 Identify limits that exist and evaluate them

1. $$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y}$$

2. $$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

3. $$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^2}$$

Problem 14 Identify all removable singularities of the functions

1. $$f(x,y) = \begin{cases} \frac{x+y}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$$

2. $$f(x,y) = \begin{cases} \frac{x^2-y^2}{x+y} & x+y \neq 0 \\ x^2 & x = -y \end{cases}$$

Problem 15 Find the gradients of functions

1.
$$f(x, y) = \sin(\ln(x + y)\cos(xy))$$

2.
$$g(x, y) = \frac{\sqrt{x + y + z}}{1 + x^2 + y^2 + z^2}$$

Problem 16 A function $z = g(x, y)$ satisfies equation $F(x, y, g(x, y)) = 0$, where

$$F(x, y, z) = x^2 + zy + y^2 + zx^2 + z^3$$

find partial derivatives g_x, g_y as functions of x, y, z.

Problem 17

1. Find formula for a normal vector to level curves of the function $f(x, y) = x^2 + 3x + y - y^3$.

2. Find critical (extreme) points of this function, determine their type.

3. Find directional derivative of f along the vector $(1, -2)$.

Problem 18 Find the absolute maximum of the function

$$f(x, y) = x^2 - 3xy + y^2$$

in the region $x^2 + y^2 \leq 1$

Problem 19 Sketch the region of integration R and switch the order of integration in the following integrals

1. $$\int_0^4 \int_0^{y^2} f(x,y)dxdy$$

2. $$\int_1^4 \int_{-\ln(x)}^{\ln(x)} f(x,y)dydx$$

3. $$\int_2^3 \int_{2-y}^{\frac{1}{y}} f(x,y)dxdy$$

Problem 20

1. Evaluate
$$\iint\limits_R e^{-x-y}dxdy$$
where R is the region in the first quadrant in which $x + y \leq 1$

2. Evaluate
$$\int_0^8 \int_{x^{\frac{1}{3}}}^2 \frac{dydx}{1+y^4}$$
(Hint:change the order of integration first.)

Problem 21 Find the mass and center of mass of the triangle with the vertices $(0, 0)$, $(1, 0)$ and $(1, 2)$ whose density is given by $\rho(x, y) = x^2$.

Problem 22 The integral

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}dydx$$

is given in orthogonal coordinates. Change it to polar coordinates.

Problem 23 Use polar coordinates to set up the integral for the volume of the solid inside the sphere $x^2 + y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 4$.

Problem 24 Find the area of the part of hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.