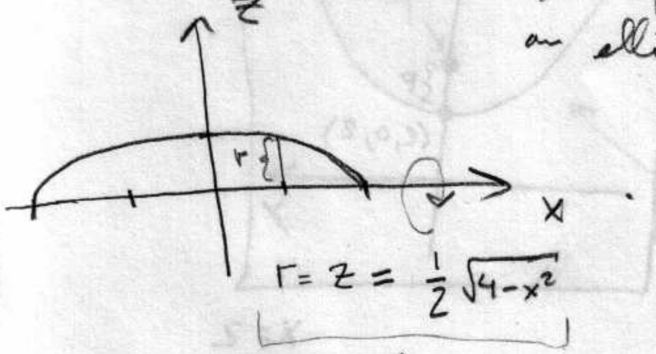
Math 203 HW3

Sec. 11.6

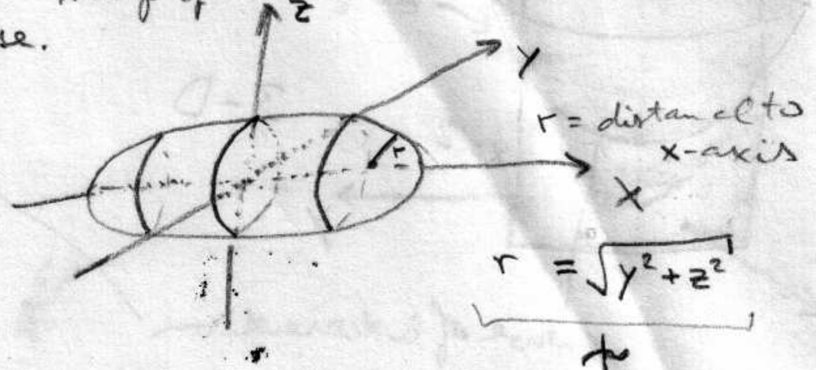
#50.

$2z = \sqrt{4-x^2}$ $\quad 4z^2 = 4 - x^2 \quad \frac{x^2}{4} + \frac{z^2}{1} = 1$

the upper half of an ellipse.



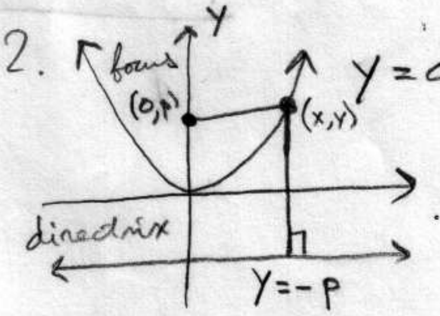
$\underbrace{r = z = \frac{1}{2}\sqrt{4-x^2}}_{\star}$



$\underbrace{r = \sqrt{y^2+z^2}}_{\dagger}$

$\star + \dagger$: $\left.\begin{aligned} \frac{1}{2}\sqrt{4-x^2} &= \sqrt{y^2+z^2} \\ 4 - x^2 &= 4y^2 + 4z^2 \end{aligned}\right\}$ $\boxed{\frac{x^2}{4} + y^2 + z^2 = 1}$

#62.



$y = ax^2$

Recall that a parabola is the set of points equidistant from a given point (the focus) and a given line (the directrix).

If the focus is at $(0,p)$ and the vertex of the parabola is at $(0,0)$, the directrix must be $y=-p$.

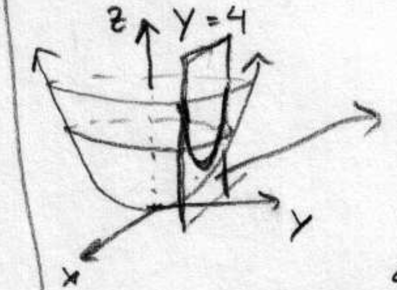
The relationship between the number p and the coefficient a in $y = ax^2$ can be seen as follows:

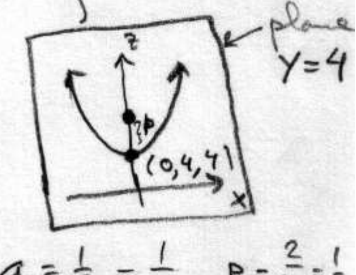
distance from (x,y) to $(0,p)$ $= \sqrt{x^2+(y-p)^2} = y + p$ = distance from (x,y) to $y=-p$

$x^2 + y^2 - 2yp + p^2 = y^2 + 2yp + p^2$

$x^2 = 4yp$ or $y = \frac{1}{4p}x^2$. so $a = \frac{1}{4p}$.

Now: a) $\left.z = \frac{1}{2}x^2 + \frac{1}{4}y^2\right\}$ $\quad z = \frac{1}{2}x^2 + 4$





$a = \frac{1}{2} = \frac{1}{4p}$. $p = \frac{2}{4} = \frac{1}{2}$

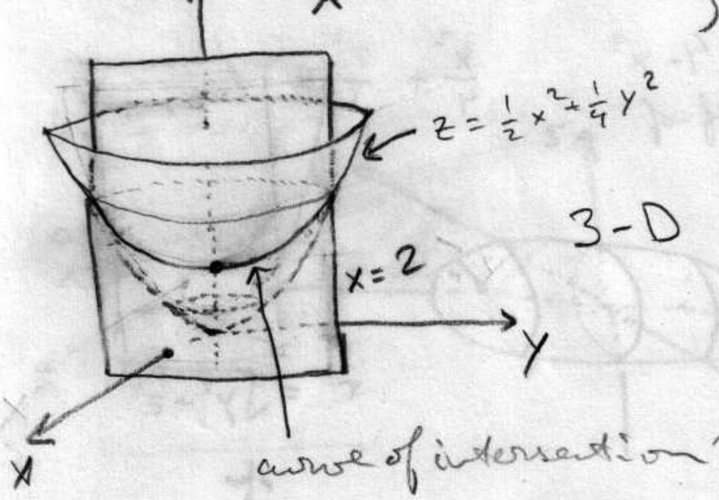
so coordinates of the focus are:

$(0, 4, 9/2)$

↑ the vertex $(0,4,4)$ shifted up in the z-direction by $p = 1/2$.

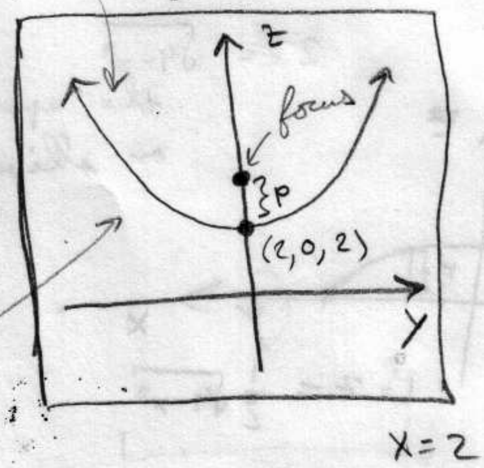
$a = \frac{1}{4}$

#62. b. $\left.\begin{array}{l} z = \frac{1}{2}x^2 + \frac{1}{4}y^2 \\ X = 2 \end{array}\right\} \quad z = \frac{1}{4}y^2 + 2$



3-D

2-D



$$p = \frac{1}{4a} = 1$$

so the coordinates of the focus are $\boxed{(2, 0, 3)}$.