Math 203 HW2

Sec. 11.3

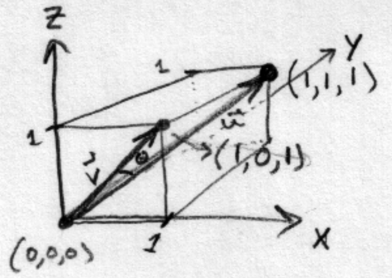
#50. $\vec{u} = \hat{\imath} + 4\hat{k}$, $\vec{v} = 3\hat{\imath} + 2\hat{k}$

a) $\text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v} = \dfrac{(1\cdot 3 + 4\cdot 2)}{(3^2+2^2)}(3\hat{\imath}+2\hat{k}) = \dfrac{11}{13}(3\hat{\imath}+2\hat{k}) = \boxed{\dfrac{33}{13}\hat{\imath} + \dfrac{22}{13}\hat{k}}$

b) component of $\vec{u}$ orthogonal to $\vec{v}$ $= \vec{u} - \text{proj}_{\vec{v}}\vec{u} = (\hat{\imath}+4\hat{k}) - \left(\dfrac{33}{13}\hat{\imath} + \dfrac{22}{13}\hat{k}\right) =$

$= \boxed{\dfrac{-20}{13}\hat{\imath} + \dfrac{30}{13}\hat{k}}$

#80.



$\vec{u} = (1,1,1)$

$\vec{v} = (1,0,1)$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{2}{\sqrt{3}\cdot\sqrt{2}} = \dfrac{2}{\sqrt{6}} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$

$\boxed{\theta = \text{Arccos}\left(\sqrt{6}/3\right)}$

Sec. 11.4

#34. $\vec{u} = \overrightarrow{AB} = (0,1,2) - (2,-3,4) = (-2,4,-2)$

$\vec{v} = \overrightarrow{AC} = (-1,2,0) - (2,-3,4) = (-3,5,-4)$

$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & 4 & -2 \\ -3 & 5 & -4 \end{vmatrix} = \hat{\imath}(-16+10) - \hat{\jmath}(8-6) + \hat{k}(-10+12) = (-6,-2,2)$

Area $= \frac{1}{2}\|\vec{u}\times\vec{v}\| = \frac{1}{2}\sqrt{36+4+4} = \frac{1}{2}\sqrt{44} = \boxed{\sqrt{11}}$

Sec. 11.5

#42. $(x-0, y+1, z-4)\cdot\vec{n} = 0$, but $\vec{n} = \hat{k} = (0,0,1)$ so: $(x, y+1, z-4)\cdot(0,0,1) = 0$

or $z - 4 = 0$ or $\boxed{z = 4}$

#112. The standard equation of a plane in 3-space is: $ax + by + cz = d$. The numbers a, b, c are the components of a vector $\vec{N} = (a, b, c)$ which is orthogonal to the plane. Given a point (x_0, y_0, z_0) on the plane, and a vector $\vec{N}$ orthogonal to it, the numbers a, b, c are the components of $\vec{N}$, and $d = ax_0 + by_0 + cz_0$. Or:
$(x-x_0, y-y_0, z-z_0)\cdot\vec{N} = 0$ is another way to write the equation.