

Practice Final Exam
MAT 131
December 2017

No notes, books or calculators.

You must show your reasoning, not just the answer.

Answers without justification will get only minimal partial credit.

Please cross out anything that is not part of your solution —
e.g., some preliminary computations that you didn't need.

Answers should be simplified if possible (e.g., $\sin(0)$ should be simplified to 0). However, do not round numerical answers unless the problem explicitly asks for it (e.g., do not replace $1/3$ by 0.33)

1. (20 pts) Compute the following limits. If the limit does not exist, please specify whether the limit is equal to ∞ , $-\infty$, or does not exist even allowing for infinite values.

(a) $\lim_{x \rightarrow 1^-} \frac{x^2 + 1}{x^2 - 1}$

(b) $\lim_{x \rightarrow 0} \frac{\sin(x)}{2^x - 1}$

(c) $\lim_{h \rightarrow 0} \frac{\sqrt{3+h} - \sqrt{3-h}}{h}$

(d) $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{3x^2 + 17x + 219}$

(e) $\lim_{x \rightarrow \infty} \frac{\sin(x^2)}{x^2 + 5}$

2. (30 pts) Compute the derivatives of the following functions.

(a) $f(x) = 2^x x^{3/2}$

(b) $f(x) = \tan(x^4 + \cos x)$

(c) $f(x) = \frac{x^2 + 7x}{(x+1)(x-3)}$

$$(d) f(x) = \arctan \sqrt{x^2 - 1}$$

$$(e) g(u) = \frac{1}{u^2} + \sqrt{u} + u^3$$

$$(f) f(t) = (\cos t)^{\tan t}$$

3. (20 pts) Let F be the function

$$F(x) = \frac{x^2}{x^3 + 4}.$$

(a) Find $\lim_{x \rightarrow \infty} F(x)$ and $\lim_{x \rightarrow -\infty} F(x)$.

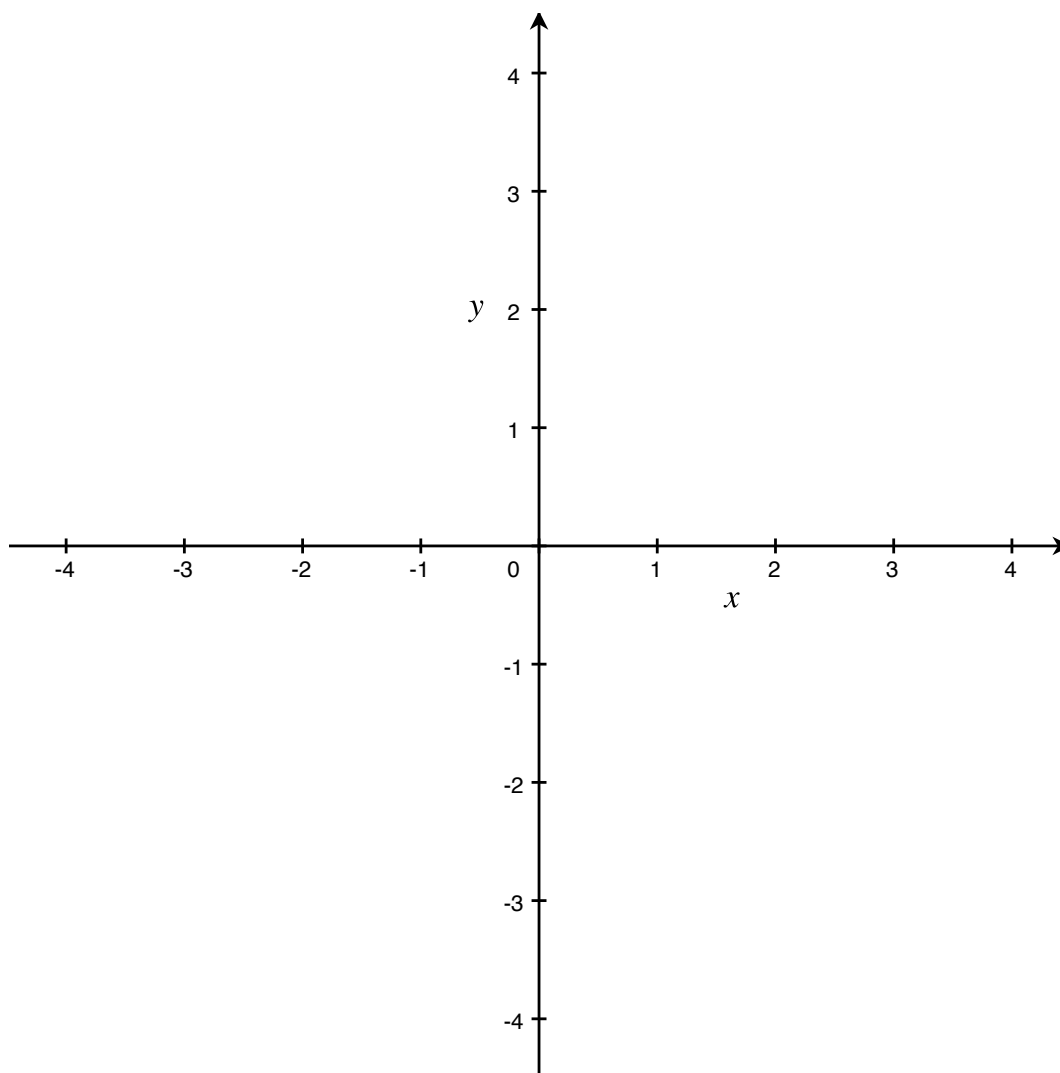
(b) The domain of F consists of all real numbers except one number a . What is a ?
Find $\lim_{x \rightarrow a^-} F(x)$ and $\lim_{x \rightarrow a^+} F(x)$.

(c) Find all the local extreme values of F and where they occur. For each of these values, specify whether it is a local maximum or a local minimum.

(d) The second derivative of F is $F''(x) = \frac{2(x^6 - 28x^3 + 16)}{(4 + x^3)^3}$.

Show that the graph of F has an inflection point between $x = 0$ and $x = 1$.

- (e) On the axes below, sketch the graph of F . Label its asymptotes and the local extreme values of F .



4. (15 pts) A TV camera is positioned 600 meters away from the rocket launch pad. The rocket is launched, rising vertically up, and the camera is kept aimed at the rocket, tracking its motion. At the moment the rocket's altitude is 600 meters, its speed is 100 m/s. How fast is the camera's angle of elevation changing at this moment? (The answer should be written in rad/s).

5. (10 pts) Find a linearization of the function $f(x) = e^{-x} \sin(\pi x)$ near the point $x = 2$.

- 6.** (15 pts) Maximize the area of a rectangle that can be inscribed in the ellipse

$$\frac{x^2}{4} + y^2 = 1$$

[Hint: instead of maximizing the area A , you can maximize A^2]

7. (20 pts) Compute each of the following (definite or indefinite) integrals.

(a) $\int_0^2 (x^3 + 2x + 1) dx$

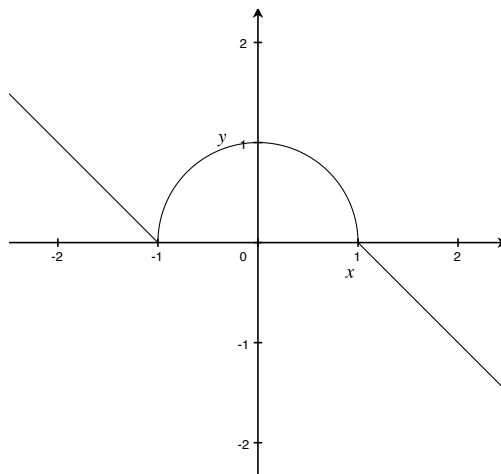
(b) $\int (3e^x + 2 \sin x) dx$

(c) $\int \sqrt{3x + 1} dx$

(d) $\int_{-\pi}^{\pi} x \cos\left(\frac{1}{x}\right) dx$

(e) $\int_0^{\sqrt{\pi}} x \sin(x^2) dx$

8. (10 pts) Below is the graph of a continuous function f . The graph is composed of the upper half of the unit circle and two half-lines with slope -1 .



- (a) Compute $\int_0^2 f(x) dx$. (Hint: You do not need to find an explicit antiderivative for f .)

- (b) Define $g(x) = \int_{-3}^{x^2} f(t) dt$. What are the critical points of g ? For each critical point c , determine whether $g(c)$ is a local minimum, a local maximum, or neither. (Note: Critical points are also called “critical numbers”.)

9. (10 pts) Write the first iteration of the Newton’s method for the problem of finding a solution of $\log(x) + 10 = x^2$.

10. (10 pts) Find the area enclosed by the x -axis, the graph of $y = \sin(x)$, y -axis and $x = 2\pi$.