

MAT125A Fall 2014

Practice Midterm I

The actual exam will consist of six problems.

Problem 1

Suppose $f(x)$ is a linear function such that $f(-1) = -1$, $f(3) = 6$. Then $f(1) = ?$

a) $3/4$, **b)** $3/2$, **c)** $5/2$, **d)** $-3/4$, **e)** 2 , **f)** none of these.

Problem 2

Express the following function as an explicit formula: take a number, take a cube of it, add one to the result, multiply the result by 3, take the logarithm base 3

a) $\log_3(3x^3 + 1)$, **b)** $3 \log_3(x^3 + 1)$, **c)** $1 + \log_3(x^3 + 1)$, **d)** $\log_3(x^3 + 3)$, **e)** $\ln(x + 1)$, **f)** none of these.

Problem 3

The function $x^2 + 1$ is a composition of the form $f(g(x))$ where

a) $f(x) = x^2$ and $g(x) = 1$, **b)** $f(x) = x^2$ and $g(x) = x + 1$, **c)** $f(x) = x + 1$ and $g(x) = x^2$, **d)** $f(x) = x$ and $g(x) = x^2$, **e)** $f(x) = (x + 1)^2$ and $g(x) = -2x$, **f)** none of these.

Problem 4

Which of the following functions is not defined for all real numbers

a) $f(x) = \cos(x^2)$, **b)** $f(x) = \ln(x^2 + 1)$, **c)** $f(x) = e^{\cos(x)}$, **d)** $f(x) = \ln(\sin(x))$, **e)** $f(x) = \sqrt{1 + \sin(x)}$, **f)** none of these.

Problem 5

Sketch

1. $y = -\frac{1}{4}x + 2$

2. $y = -x^2 + 2x + 1$.

3. $y = |1 + \frac{1}{x}|$

Solution of part two Use $-x^2 + 2x + 1 = -x^2 + 2x - 1 + 2 = -(x - 1)^2 + 2$. Start with graph of standard parabola $y = x^2$ and apply reflection $x^2 \rightarrow -x^2$ and two shifts $-x^2 \rightarrow -(x - 1)^2$ and $-(x - 1)^2 \rightarrow -(x - 1)^2 + 2$.

Problem 6

Find each of the following limits (or say that it doesn't exist)

1. $\lim_{x \rightarrow -1} \frac{x^2 - 6x - 7}{x - 7}$
2. $\lim_{x \rightarrow 7} \frac{x^2 - 6x - 7}{x - 7}$
3. $\lim_{x \rightarrow 0} \frac{|x|^3}{x^3}$
4. $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x}$, $a > 0$ is an arbitrary real number.
5. $\lim_{s \rightarrow 0} s^2 \cos(s + 1/s)$.

Problem 7

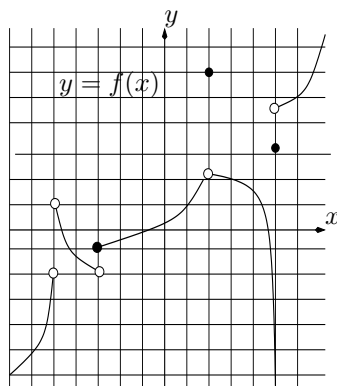
Answer the following questions based on the graph of the function f below. Assume that the domain of the function is the set $-7 \leq x < -5$ and $-5 < x \leq 7$.

1. What $f(-3)$
2. Does $\lim_{x \rightarrow -3^-} f(x)$ exist? If it does, what is its value?
3. Does $\lim_{x \rightarrow -3^+} f(x)$ exist? If it does, what is its value?
4. Does $\lim_{x \rightarrow -3} f(x)$ exist? If it does, what is its value?
5. At what points f is continuous from the right but not from the left?
6. At what points f is discontinuous?

Problem 8

Consider the function

$$f(t) = \begin{cases} \frac{t}{t-1} & t \geq 0, t \neq 1 \\ t + 1 & t < 0 \end{cases}$$



1. At which points is this function continuous?
2. Find the left and right limits, if they exist, at $t = 0$.

Problem 9 Explain, without using a graphing calculator, why the equation $x^5 = 3x - 1$ must have a solution with $0 < x < 1$.

Problem 10 A package of spinach in New York City has 100 *E Coli* bacteria, and the number of bacteria in the spinach triples every hour.

1. Give a formula $E(t)$ for the number of bacteria in the spinach after t hours.
2. How many bacteria are present after 4 hours?