

# MAT125A Fall 2014

## Practice Midterm II

The actual exam will consist of six problems.

### Problem 1

Compute the following limits. Please distinguish between “limit is equal to  $\infty$ ”, “limit is equal to  $-\infty$ ” and “the limit doesn’t exist even allowing for infinite values”:

1.  $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{2x^3 - 15x}$
2.  $\lim_{x \rightarrow 2^-} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$
3.  $\lim_{x \rightarrow 3^+} \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$
4.  $\lim_{x \rightarrow \infty} \frac{1}{e^{(x^2)} + 1}$
5.  $\lim_{x \rightarrow -1^+} \tan^{-1}\left(\frac{1-x}{1+x}\right)$
6.  $\lim_{x \rightarrow +\infty} \frac{\sin^{-1}\left(\frac{1-x}{1+x}\right)}{\cos^{-1}\left(\frac{1-x}{1+x}\right)}$
7.  $\lim_{x \rightarrow 0} \frac{\sin(2x)^2}{x^2}$

**Problem 2** Find horizontal and vertical asymptotes of

1.  $y = \frac{x^3 + 2x + 1}{2x^3 - 32x}$
2.  $y = \frac{x^2 - 2x - 3}{x^2 - 5x + 6}$
3.  $y = \frac{1}{e^{x^2} + 1}$
4.  $y = \frac{\sin(2x)^2}{x^2}$

### Problem 3

Find the derivative of each function

1.  $\frac{1}{x^5}$

2.  $\cos(x)$
3.  $e^x$
4.  $x^2e^x$
5.  $\tan(1 + x^2)$
6.  $(\sin(x))^2$
7.  $\cos(xe^x)$
8.  $\frac{\cos(x^2)}{x^2}$
9.  $\sqrt{\sin(\frac{1}{x^2})}$
10.  $\frac{1}{\sin^{-1}(x)}$
11.  $\tan^{-1}(\sqrt{x})$

**Problem 4**

1. Compute  $\sin(\tan^{-1}(2))$
2. Simplify  $\cos(2 \sin^{-1}(x))$

**Problem 5**

Find an equation for the tangent line to the curve

$$y = \sin(x) - 5x + 3 \text{ at the point } (0, 3)$$

**Problem 6**

Find the first and second derivatives of the following functions.

1.  $f(x) = x^4 + 7x + 3e^{x^3}$
2.  $g(x) = \cos(x^4)$

**Problem 7**

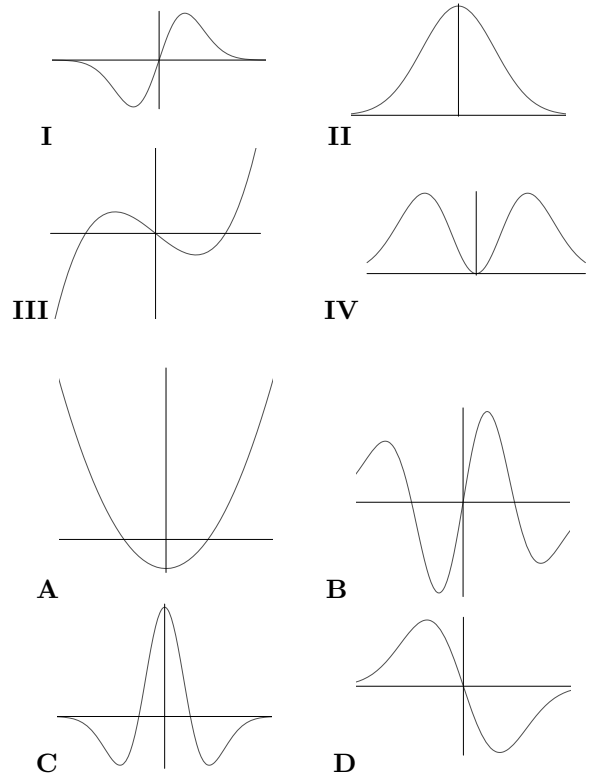
Consider the function  $y = f(x)$  defined implicitly by the equation

$$xy + x^2y^3 = 10$$

, such that  $f(1) = 2$ . Find  $f'(1)$  and  $f''(1)$ .

**Problem 8**

Match the graphs of functions **I-IV** below with the graphs of their derivatives **A-D**. (Justification is not required.)



**Problem 9**

1. Suppose  $f(x) = \sin(x)g(x)$ . Compute  $f''(\pi/3)$  if  $g(\pi/3) = g'(\pi/3) = g''(\pi/3) = 4$
2.  $F(x) = G(H(x))$ . Compute  $F'(2)$  if  $H(2) = 3, H'(2) = 9, H(3) = 2, H'(3) = 1/9, G(2) = 1/3, G'(2) = 1/9, G(3) = 2, G'(3) = 9,$