1. (a) $\lim_{x\to -1} x^2+x-1 = -1$

(b) $\lim_{x\to -3} \frac{x^2+2x-3}{x+3} = \lim_{x\to -3} \frac{(x+3)(x-1)}{x+3} = \lim_{x\to -3} x-1 = -4$

(c) $\lim_{t\to 0} \frac{\sqrt{2-t}-\sqrt{2}}{t} = \lim_{t\to 0} \frac{2-t-2}{t(\sqrt{2-t}+\sqrt{2})}$

$= \lim_{t\to 0} \frac{-1}{\sqrt{2-t}+\sqrt{2}} = \frac{-1}{2\sqrt{2}}$

(d) $\lim_{x\to 0} x\sin\left[\pi\left(x^2+\frac{1}{x^2}\right)\right] = 0$

$\left(-1 \le \sin\left[\pi\left(x^2+\frac{1}{x^2}\right)\right] \le 1\right)$

(e) $\lim_{x\to\infty} \frac{x^3+2x+1}{x^3-2x+1} = \lim_{x\to\infty} \frac{1+2\frac{1}{x^2}+\frac{1}{x^3}}{1-2\frac{1}{x^2}+\frac{1}{x^3}} = 1$

(f) $\lim_{x\to \pi/2} \frac{\cos x}{2x-\pi} = \lim_{x\to\pi/2} \frac{-\sin x}{2} = -\frac{1}{2}$

(g) $\lim_{x\to 0} \frac{\ln(\cos x)}{\cos x} = \frac{0}{1} = 0$

(h) $\lim_{x\to 0} \frac{\ln(\cos x)}{\sin x} = \lim_{x\to 0} \frac{\frac{1}{\cos x}\cdot \sin x}{\cos x} = 0$

(i) $\lim_{x\to+\infty} (x-\ln(x)) = \lim_{x\to+\infty} \ln e^x - \ln x = \lim_{x\to+\infty} \ln\frac{e^x}{x} = \lim_{x\to+\infty} \ln e^x = +\infty$

(j) $\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \left(\frac{\sin x}{x}\right) x\ln x$

$= \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -x = 0$

(k) $\lim_{x\to 0}\left(\frac{\cos x}{\cos(2x)}\right)^x = \lim_{x\to 0} 1^x = 1$

(l) $\lim_{x\to 0^+} x^{\frac{1}{x}} = \lim_{x\to 0^+} e^{\frac{\ln x}{x}} = \lim_{x\to 0^+} e^{\frac{x\ln x}{x^2}} = \lim_{x\to 0^+} e^{\frac{1+\ln x}{2x}}$

(m) $\lim_{x\to 0^+} \tan(x)^{5x} = 1$

(n) $\lim_{x\to+\infty}\left(\frac{1+x}{2+x}\right)^x = \lim_{x\to+\infty} e^{x\ln\left(\frac{1+x}{2+x}\right)}$

$= \lim_{x\to+\infty} e^{\frac{\ln\left(\frac{1+x}{2+x}\right)}{1/x}}$

$= \lim_{x\to+\infty} e^{\frac{\frac{1}{1+x}-\frac{1}{2+x}}{-\frac{1}{x^2}}} = \lim_{x\to+\infty} e^{-x^2\left(\frac{2+x-1-x}{(1+x)(2+x)}\right)}$

$= \lim_{x\to+\infty} e^{-x^2\left(\frac{1}{2+3x+x^2}\right)} = e^{-1} = \frac{1}{e}$

(l) $\lim_{x\to 0^+} x^{\frac{1}{x}} = 0$

2. (a) $f(x) = x^3 - 12x^2 + x + 2\pi$

$f'(x) = 3x^2 - 24x + 1$

(b) $f(x) = (2x+1)\sin x$

$f'(x) = 2\sin(x) + (2x+1)(-\cos x)$

(c) $g(s) = \sqrt{1+e^{2s}}$

$g'(s) = \dfrac{2e^{2s}}{2\sqrt{1+e^{2s}}} = \dfrac{e^{2s}}{\sqrt{1+e^{2s}}}$

(d) $h(t) = \dfrac{1+e^t}{1-e^t}$

$h'(t) = \dfrac{(1-e^t)e^t - (1+e^t)(-e^t)}{(1-e^t)^2} = \dfrac{e^t - e^{2t} + e^t + e^{2t}}{(1-e^t)^2}$

$= \dfrac{2e^t}{(1-e^t)^2}$

(e) $f(x) = (2x+2)^{10}$

$f'(x) = 10(2x+2)^9 \cdot 2 = 20(2x+2)^9$

(f) $g(x) = x^{\sin x}$

$\ln g = \sin x \ln x$

$\dfrac{d}{dx}[\ln g] = \dfrac{d}{dx}[\sin x \ln x]$

$\dfrac{g'}{g} = \dfrac{\sin x}{x} + \cos x \ln x$

$g'(x) = g\left(\dfrac{\sin x}{x} + \cos x \ln x\right)$

$g'(x) = x^{\sin x}\left[\dfrac{\sin x}{x} + \cos x \ln x\right]$

(g) $\tan^{-1}\left(\frac{y}{1-y^2}\right)$

$$\frac{d}{dy}\left[\tan^{-1}\left(\frac{y}{1-y^2}\right)\right] = \frac{1}{1+\left(\frac{y}{1-y^2}\right)^2} \cdot \frac{(1-y^2)-y(-2y)}{(1-y^2)^2}$$

$$= \frac{1-y^2+2y^2}{(1-y^2)^2+y^2}$$

$$= \frac{1+y^2}{1-2y^2+y^4+y^2}$$

$$= \frac{1+y^2}{1-y^2+y^4}$$

(h) $\sin^{-1}\left(\frac{1}{t^4}\right)$

$$\frac{d}{dt}\left[\sin^{-1}\left(\frac{1}{t^4}\right)\right] = \frac{1}{\sqrt{1-\left(\frac{1}{t^4}\right)^2}} \cdot \frac{-4}{t^5}$$

$$= \frac{-4t^4}{\sqrt{t^8-1}\,t^5} = \frac{-4}{\sqrt{t^8-1}\,t}$$

3. (a) xe^{-x^2}

(a) asymptotes

$xe^{-x^2} = \text{DNE}$ for no x.

$\lim_{x \to +\infty} xe^{-x^2} = 0$

$\lim_{x \to -\infty} xe^{-x^2} = 0$

(b) intercepts

$xe^{-x^2} = 0 \Rightarrow x = 0$

$y = (0)e^{-(0)^2} \Rightarrow y = 0$

$(0,0)$

(c) symmetry: odd

(d) $f'(x) = xe^{-x^2}(-2x) + e^{-x^2}$

$= e^{-x^2}(1-2x^2)$

(e) $f(x)$ increases

$f'(x) = \underbrace{e^{-x^2}}(1-2x^2) > 0$

always positive

$1-2x^2 > 0$

$\frac{1}{2} > x^2$

$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$f(x)$ decreases

$f'(x) = e^{-x^2}(1-2x^2) < 0$

$1-2x^2 < 0$

$\frac{1}{2} < x^2$

$-\frac{1}{\sqrt{2}} > x$, $x > \frac{1}{\sqrt{2}}$

$$f''(x) = \frac{d}{dx}\left[e^{-x^2}(1-2x^2)\right] = e^{-x^2}(-2x)(1-2x^2) + e^{-x^2}(-4x)$$

$$= e^{-x^2}(-2x + 4x^3 - 4x) = e^{-x^2}(4x^3 - 6x)$$

(f) local max and local min

$$f'(x) = e^{-x^2}(1-2x^2) = 0$$

$$1 - 2x^2 = 0$$

$$x = \pm\frac{1}{\sqrt{2}}$$

$$f''\left(\frac{1}{\sqrt{2}}\right) = e^{-\frac{1}{2}}\left(\frac{4}{2\sqrt{2}} - \frac{6}{\sqrt{2}}\right) < 0$$

$$f''\left(-\frac{1}{\sqrt{2}}\right) = e^{-\frac{1}{2}}\left(-\frac{4}{2\sqrt{2}} + \frac{6}{\sqrt{2}}\right) > 0$$

$x = \frac{1}{\sqrt{2}}$ local max

$x = -\frac{1}{\sqrt{2}}$ local min

(g) inflection point

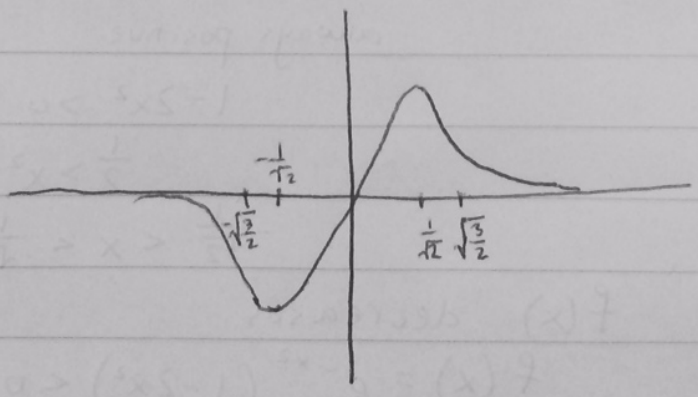
$$f''(x) = e^{-x^2}(4x^3 - 6x) = 0$$

$$4x^3 - 6x = 0$$

$$x(2x^2 - 3) = 0$$

$x = 0$ $\quad x = \pm\sqrt{\frac{3}{2}}$

(h)



(b) $\frac{x+2}{x-2}$

(a) asymptotes

$\frac{x+2}{x-2}$ = DNE when $x=2$

$\lim_{x\to+\infty} \frac{x+2}{x-2} = 1$ $\lim_{x\to-\infty} \frac{x+2}{x-2} = 1$

(b) intercept

$\frac{x+2}{x-2} = 0 \Rightarrow x+2=0$ $x=-2$

$(-2,0)$

$f(0) = \frac{2}{-2} = -1$ $(0,-1)$

(c) symmetric : no

(d) $f'(x) = \frac{(x-2)-(x+2)}{(x-2)^2} = \frac{-4}{(x-2)^2}$

(e) $f(x)$ increases

$f'(x) = \frac{-4}{(x-2)^2} > 0$ for no x

$f(x)$ decreases

$f'(x) = \frac{-4}{(x-2)^2} < 0$ for all x

$\therefore f(x)$ is a decreasing function

$$f''(x) = \frac{d}{dx}\left[\frac{-4}{(x-2)^2}\right] = \frac{(-2)(-4)}{(x-2)^3} = \frac{8}{(x-2)^3}$$

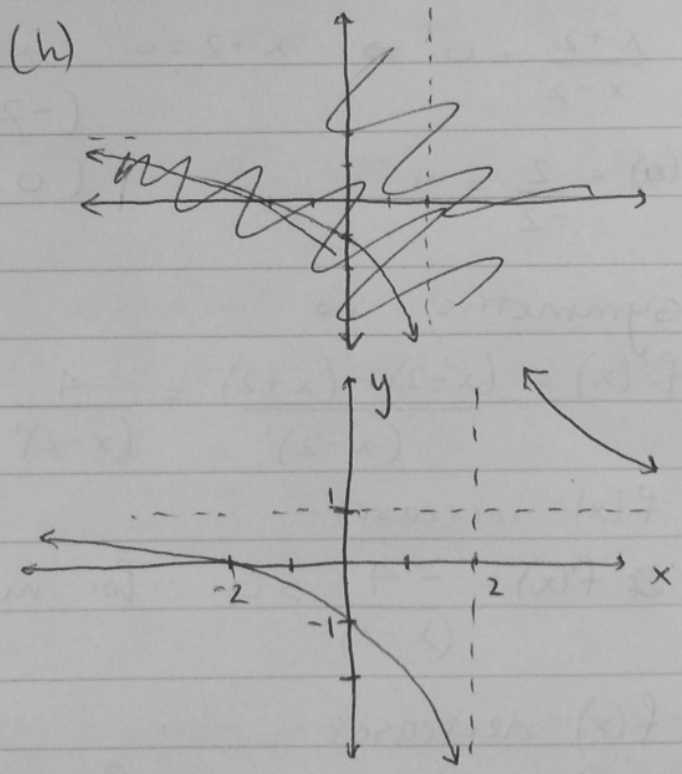
(f) local min and local max

$$0 = \frac{-4}{(x-2)^2} \quad \text{for no } x$$

(g) inflection points

$$f''(x) = \frac{8}{(x-2)^3} = 0 \quad \text{for no } x$$

(h)



(c) $\frac{x^2}{x^2-3}$

(a) asymptotes

$\frac{x^2}{x^2-3} = \text{DNE} \Rightarrow x^2-3=0 \quad x=\pm\sqrt{3}$

$\lim_{x\to+\infty} \frac{x^2}{x^2-3} = 1 \qquad \lim_{x\to-\infty} \frac{x^2}{x^2-3} = 1$

(b) intercepts

$\frac{x^2}{x^2-3} = 0 \quad \text{at} \quad x=0$

$f(0) = \frac{0^2}{0^2-3} = 0 \quad \therefore (0,0)$

(c) symmetric : even

(d) $f'(x) = \frac{(x^2-3)(2x)-x^2(2x)}{(x^2-3)^2} = \frac{2x^3-6x-2x^3}{(x^2-3)^2}$

$= \frac{-6x}{(x^2-3)^2}$

(e) $f(x)$ increases

$f'(x) = \frac{-6x}{(x^2-3)^2} > 0 \Rightarrow -6x>0 \Rightarrow x<0$

$f(x)$ decreases

$f'(x) = \frac{-6x}{(x^2-3)^2} < 0 \Rightarrow -6x<0 \Rightarrow x>0$

$$f''(x) = \frac{d}{dx}\left[\frac{-6x}{(x^2-3)^2}\right] = \frac{(x^2-3)^2(-6) - (-6x)\,2(x^2-3)(2x)}{(x^2-3)^4}$$

$$= \frac{(x^4-6x^2+9)(-6) + 24x^4 - 72x^2}{(x^2-3)^4}$$

$$= \frac{18x^4 - 36x^2 - 54}{(x^2-3)^4} = \frac{18(x^4-2x^2-3)}{(x^2-3)^4}$$

$$= \frac{18(x^2-3)(x^2+1)}{(x^2-3)^4} = \frac{18(x^2+1)}{(x^2-3)^3}$$

(f) local max and min

$$f'(x) = \frac{-6x}{(x^2-3)^2} = 0 \Rightarrow x = 0$$

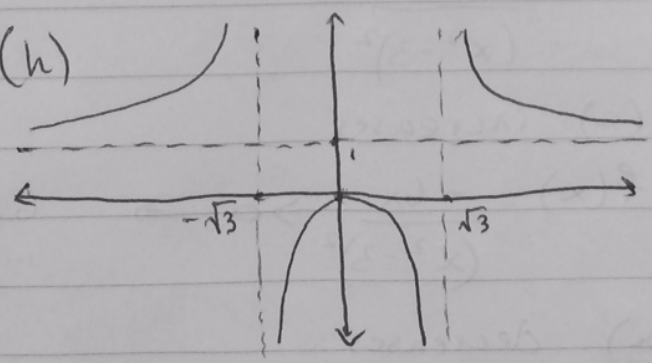
$$f''(0) = \frac{18}{(-3)^3} < 0 \qquad x=0 \text{ is local max}$$

(g) inflection points

$$f''(x) = \frac{18(x^2+1)}{(x^2-3)^3} = 0 \Rightarrow x^2+1 = 0$$

for no real x

(h)



(-1)

4. $f(x) = \dfrac{1}{\sqrt{1+x}}$

linear approximation: $f(x) \approx f(0) + f'(0)(x)$

$f(0) = 1$

$f'(x) = -\dfrac{1}{2\sqrt{(1+x)^3}}$

$f'(0) = -\dfrac{1}{2}$

$f(x) \approx 1 + \left(-\frac{1}{2}\right)(0.1) = 0.95$

6. (a) $f(x) = x^3 - x - 1$

$f(1) = -1$

$f(2) = 5$

By Intermediate Value Theorem since $f(x)$ is continious everywhere, including the interval $[1,2]$, $f(x)$ must take all the values between $f(1)$ and $f(2)$ on that interval. ~~In other words~~ Since $f(1) = -1$ and $f(2) = 5$ and the interval $[-1,5]$ includes 0 $f(x)$ must ~~take on~~ be equal to zero when $x \in [1,2]$. Therefore $f(x)$ has a ~~unique~~ root.

(b) Function $f(x) = \tan(x)$ is not continious and therefore it does not satisfy the conditions of the Intermediate Value Theorem.

8. (a) $y^2(y^2-4) = x^2(x^2-5)$

$$\frac{d}{dx}\left[y^2(y^2-4)\right] = \frac{d}{dx}\left[x^2(x^2-5)\right]$$

$$\frac{d}{dx}\left[y^4 - 4y^2\right] = \frac{d}{dx}\left[x^4 - x^2 5\right]$$

$$4y^3y' - 4 \cdot 2yy' = 4x^3 - 10x$$

$$y'(4y^3 - 8y) = 4x^3 - 10x$$

$$y' = \frac{4x^3 - 10x}{4y^3 - 8y}$$

$$y'\Big|_{(0,-2)} = \frac{4(0)^3 - 10(0)}{4(-2)^3 - 8(-2)} = 0$$

(b) $\sin y + x^2 + 4y = \cos x + \frac{\pi^2}{9} - \frac{1}{2}$

$$\frac{d}{dx}\left[\sin y + x^2 + 4y\right] = \frac{d}{dx}\left[\cos x + \frac{\pi^2}{9} - \frac{1}{2}\right]$$

$$+\cos y\, y' + 2x + 4y' = -\sin x$$

$$y'(\cos y + 4) = -\sin x - 2x$$

$$y' = \frac{-\sin x - 2x}{\cos y + 4}$$

$$y'\Big|_{(\frac{\pi}{3},0)} = \frac{-\sin\left(\frac{\pi}{3}\right) - 2\left(\frac{\pi}{3}\right)}{\cos(0) + 4} = \frac{-\frac{\sqrt{3}}{2} - \frac{2\pi}{3}}{4}$$

$$= -\frac{\sqrt{3}}{8} - \frac{2\pi}{12}$$