Problem Set #7

1. doCarmo, Chapter 3, # 2, 3, 5, 8, 9
2. Prove that a manifold, M, is orientable if and only if it's frame bundle, $F(M)$ is disconnected.
3. Let G be a Lie subgroup of $GL(n, \mathbb{R})$ which admits a bi-invariant Riemannian metric. We have seen that it is possible to identify T_eG with a subspace of $M(n, \mathbb{R})$. Prove that, under this identification, the exponential map is given by
$$\exp_e(A) = e^A = 1 + A + \frac{A^2}{2!} + \cdots$$
(Hint: You might want to do problem 2 of doCarmo first.)
4. Let S be a two dimensional Riemannian manifold. Use the exponential map to prove that, around each point $p \in S$, there is a parameterization of S, $\mathbf{x} : U \to S$, such that in local coordinates the metric is given by the matrix
$$g_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & f \end{pmatrix}$$
for some real function f.