§ 2.5 The change of coordinate matrix

THM 2.22 $Q := [I_V]_{\beta'}^{\beta}$. (a) Q inv. (b) $[v]_\beta = Q[v]_{\beta'}$.

- $x'_j = \sum_i Q_{ij} x_i$ ($\beta' = \beta Q$).
- Q change of coordinate matrix, changing β'-coord into β coordinates.
- $Q_{ij} = ?$ Answer: the j^{th} column is $[x'_j]_\beta$, the β-coordinates of x'_j
- Q^{-1} changes β-coord into β'-coord.

Ex 1 R^2, $\beta =$, $\beta' =$. Finds Q. Finds $[(2,4)]_\beta$ by using Q & $[(2,4)]_{\beta'}$

Summary: Thm 2.22 introduces the change of coord matrix, tells me it is invertible, (and Exc. 11 tells me the inverse is also a change of coord. matrix), and it tells me how to express the β-coordinates of any vector ~~in the~~ by using the matrix Q and the β'-coordinates (which I need to find first).

THM 2.23 $T: V \to V$. β, β'. $[T]_{\beta'} = Q^{-1}[T]_\beta Q$.

Ex 2 $T: R^2 \to R^2$; β & β' as in Ex 1. Find $[T]_\beta$. Find Q (see Ex 1).
It asks you to verify $Q^{-1} = \ldots$, It asks you to calculate $Q^{-1}[T]_\beta Q$.
It asks you to verify $Q^{-1}[T]_\beta Q$ is $[T]_{\beta'}$ by calculating $[T]_{\beta'}$ in the usual way.

Ex 3 Uses thm 2.23 to find $[T]_{\beta = std}$ where T is the reflection about the line $y = 2x$. Use $\beta' = \left\{\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}-2\\1\end{pmatrix}\right\}$. Then $[T]_{\beta'} = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$.
$Q = \begin{pmatrix}1 & -2\\2 & 1\end{pmatrix}$. Thm 2.23 $\Rightarrow$ $[T]_{\beta'} = Q^{-1}[T]_\beta Q$, so $[T]_\beta = Q[T]_{\beta'}Q^{-1}$ which is...

Corollary $A \in M_{n\times n}$, γ. Then $[L_A]_\gamma = Q^{-1}AQ$, Q has j^{th} col, j^{th} vector of γ.

Ex 4 Computes $[L_A]_\gamma$ for a given A 3×3, and a γ.

Summary Thm 2.23 expresses the matrix of T in β' via the matrix of T in β & the change of coordinate basis Q (from β' to β).

Caution: $Q^{-1}[T]_\beta Q \neq Q[T]_\beta Q^{-1}$!

The corollary does the same, but for those $T: F^n \to F^n$ of the form $T = L_A$ (A a given $n\times n$ matrix).

Definition B is similar to A if $\exists$ Q inv s.t. $B = Q^{-1}AQ$.

N.B.: In Thm 2.23, $[T]_{\beta'}$ & $[T]_\beta$ are similar.

In the corollary, A & $[L_A]_\gamma$ are similar.

Similarity is an equivalence relation.

Exc

1 Word out complete answers. If applicable, give examples, counterexamples.

2 Find Q, from β' to β [solve linear system of equations]

3 Similar to 2, but $P_2(R)$ instead of R^2. (do few)

4 Find $[T]_{\beta'}$ (first $[T]_\beta$, then Q, then Q^{-1}...)

5 Like 3 but for $P_1(R)$

6 Find $[L_A]_\beta$ directly & also via $[L_A]_\beta = Q^{-1}AQ$.

7 (a) use Ex 2; (b) find the right basis: $\beta' = \{(1,m),(-m,1)\}$...

8 $\underset{\beta,\beta'}{T: V} \to \underset{\gamma,\gamma'}{W}$. Then $[T]_{\beta'}^{\gamma'} = P^{-1}[T]_\beta^\gamma Q$, $Q: \beta'$ to β, $P: \gamma'$ to γ.

IMPORTANT Exc 8 is "Theoretical"; you need to imitate the proof of Thm 2.23 in this more complicated case.

9 similarity is an equiv. relation (remind yourself what this means) (Theoretical)

10 A & B similar $\Rightarrow tr(A) = tr(B)$. (Theoretical)

11 $Q: \alpha$ into β, $R: \beta$ into γ, then $RQ: \alpha$ into γ. $Q^{-1}: \ldots$? [Theoretical]

12 Prove a corollary [Theoretical]

13 An invertible matrix and a basis give a new basis and the matrix of the change of bases is...

14 Theoretical...

Some sections have 20 or more exercises. The first time around, do the first 10, or so. See if they cover all the material in the section. If not, seek out more problems so the whole section is covered. Eventually, you should try all of them. Some of them are pretty theoretical; one example is # 14; often, these are late in the exercise section; do not ignore them: for example, something like §2.6, # 20 (T, T^t), could occur in a test.