MAT 205 REVIEW SHEET

CHAPTER 9 VECTORS AND THE GEOMETRY OF SPACE

9.1 Three dimensional coordinate system: coordinate planes; octants; distance; sphere.

Typical problems: examples 1-6; exercises: 1-12,14,16,17-30.

9.2 Vectors: two and three dimensional vectors; representation of a vector; vector from A to B; length; addition of vectors; multiplication by a scalar; parallel; difference; properties; **i**, **j**, **k**; unit vectors.

Typical problems: examples 1-4; exercises: 1-18.

9.3 The dot product: dot product; perpendicular; component form; properties; vector projection; scalar projection; component of **b** along **a**.

Typical problems: examples 1, 3-7; exercises: 1-26.

9.4 The cross product: the right-hand rule; cross product; parallel; properties; determinants 2×2 and 3×3; component form; triple products; volume of parallelepiped; vector triple product.

Typical problems: examples 2-6; exercises: 1-4,7-16;18-24.

9.5 Equations of lines and planes: vector equation, direction numbers and symmetric equations for lines in space; vector, scalar and linear equations of the plane; parallel planes; angle between planes; point-plane distance.

Typical problems: examples 1-10; exercises: 1-44..

9.6 Functions and surfaces: domain and range; independent and dependet variables; graphs.

Typical problems: examples 1-8; exercises: 1-24.

CHAPTER 10 VECTOR FUNCTIONS

10.1 Vector functions and space curves: vector function of one variable; limits; continuity; parametric equations.

Typical problems: examples 1-5; exercises: 1-18,27-30.

10.2 Derivatives integrals: derivative of a vector valued functions of one variable; tangent vector; tangent line; rules of differentiation; integrals $\int_a^b \mathbf{r}(t)\,dt$ and fundamental theorem of calculus.

Typical problems: examples 1-6; exercises: 3-8,9-22,29-36.

10.3 Arc length: how to compute the arc lenght from any parametrization; arc length function; parametrization with respect to arc length.

Typical problems: examples 1-2; exercises: 1-5,7-10.

10.4 Motion in space: position vector; velocity vector; acceleration vector; finding position etc given initial conditions; projectile.

Typical problems: examples 1-5; exercises: 2-7,9-12.

10.5 Parametric surfaces: parametric surfaces; parametric equations; grid curves.

Typical problems: examples 1,3-7; exercises: 1-4,9-22.

CHAPTER 11 PARTIAL DERIVATIVES

11.1 Functions of several variables: domain, range, graph, level curves (what are they, can you draw them, how do you find them, level curves=z-traces).

Typical problems: examples 3-9,11-12; exercises: 5-22.

11.2 Limits and continuity: limit at a point, limit at a point along a path, checking different paths to prove there is no limit, continuity at a point and on a domain.

Typical problems: examples 1-9; exercises: 1-18,21-22,25-32.

11.3 Partial derivatives: partial with respect to x at (a,b) (it is a number), partial with respect to x at a variable point (x,y) (it is a function of (x,y)); same for the other variables; what is the meaning of the partial derivative at a point (a,b) and how to read it on the graph?; higher derivatives; mpre variables.

Typical problems: examples 1-9; exercises: 11-34,37-50.

11.4 Tangent planes and linear approximation: tangent plane to a surface $z = f(x, y)$ (the surface is the graph of the function) at a point $P = (x_0, y_0, z_0))$; how can you draw it?; linearization of f at (a, b) (also in three variables); linear approximation; what does it mean for aa function to be differentiable?; when is a function differentiable?; differentials; total differentials; what are they and how do you picture them?; tangent planes to parametric surfaces.

Typical problems: examples 1-2,4-7; exercises: 1-4,9-12,15,19-24,33-36.

11.5 The chain rule: case 1, case 2, general version.

Typical problems: examples 1,3-7; exercises: 1-19.

11.6 Directional derivatives and the gradient vector: what is a directional derivative, how do you compute it, what is its meaning in a picture? If f is differentiable how do you compute the directional derivatives at a point? What is the gradient vector? How does it relate to the directional derivatives? What is the meaning of it? How do you maximize directional derivatives? How do you find the tangent plane to a level surface?

Typical problems: examples 1-6,8; exercises: 1-18,21-23,27-30.

CHAPTER 12 MULTIPLE INTEGRALS

12.1 Double integrals over rectangles: Double integrals over a rectangle;the volume under the graph of a positive function over a rectangle is given by a double integral; by subdividing the rectangle and choosing sample points (for example the midpoint rule) you can approximate the double integral; average value of a function over a rectangle; properties of the double integral.

Typical problems: examples 1-3; exercises: 1-5,11-12,15-16.

12.2. Iterated integrals: what is an iterated integral over a rectangle?; what is Fubini's Theorem and why is it important? Two different orders of integration, but the same answer!

Typical problems: examples 1-5; exercises: 1-23.

12.3 Double integrals over general regions: Use rectangular regions to define double integrals over a general region; regions of type I; type II; properties of double integrals;

Typical problems: examples 1-6; exercises: 1-20,25-38.

12.4 Double integrals in polar coordinates: polar rectangles; change to polar coordinates in a double integral ($r\,dr$!!); polar regions and integrals over polar regions.

Typical problems: examples 1-3; exercises: 1-19,23-26.

12.6 Surface area: Always a double integral; for parametric surfaces; special easier case: the surface is a graph of a function $z = f(x, y)$.

Typical problems: examples 1-2; exercises: 1-9.

12.7 Triple integrals: over a box; iterated triple integrals (six!); Fubini's theorem; over a general bounded region in space; type 1,2,3.

Typical problems: examples 1-4; exercises: 1-18,21-30.

CHAPTER 13 VECTOR CALCULUS

13.1 Vector fields: definition on the plane and in space; force fields; gradient vector fields; conservative fields; potential functions.

Typical problems: examples 1-6; exercises: 1-18,21-24.

13.2 Line integrals: $\int_C f(x,y,z)\,ds$; $\int_C f(x,y,z)\,dx$, $\int_C f(x,y,z)\,dz$, $\int_C P\,dx + Q\,dy + R\,dz$; curves made of several pieces; orientation; $\int_{-C} f\,dx = -\int_C f\,dx$; $\int_C \mathbf{F}\cdot d\mathbf{r}$; connection between line integrals of vector fields and line integrals.

Typical problems: examples 1-2,4-8; exercises: 1-18.

13.3 The fundamental theorem for line integrals: the fundamental theorem for line integrals; on the plane $P_y = Q_x$ implies conservative; finding potential functions.

Typical problems: examples 1-5; exercises: 3-18,21-22.

13.4 Green's theorem: Green's theorem; in reverse; to compute areas; regions with holes.

Typical problems: examples 1-5; exercises: 1-2,5-19.

13.5 Curl and divergence: curl; symbolic expression; $curl(\nabla f) = 0$; test for not conservative; test for conservative; divergence; $div\mathbf{F} = \nabla\cdot\mathbf{F}$; $div\,curl\mathbf{F} = 0$; Green's theorem vector form.

Typical problems: examples 1-5; exercises: 1-20.

13.6 Surface integrals: how to compute $\int\int_S f(x,y,z)\,dS = \int\int_D f(r(u,v))|r_u\,xr_v|\,dA$; special case: graphs; oriented surfaces; surface integrals of vector fields; flux; surface integral of a vector field over a surface $\int\int_S \mathbf{F}\cdot d\mathbf{S}$;

Typical problems: examples 1-6; exercises: 5-25.

13.7 Stoke's theorem: Stoke's Theorem.

Typical problems: examples 1-2; exercises: 1-11,13-15.

13.8 The divergence theorem: the divergence theorem.
Typical problems: examples 1-2; exercises: 3-12.

LIST OF KIND OF INTEGRALS

$\int_a^b \mathbf{r}(t)\,dt$. It is a vector. Moslty computed via direct integration.

$\int\int_D f(x,y)\,dA$. Double int over a region in the plane. Mostly computed using Fubini's theorem on iterated integrals and type I, II or polar. $A(D) = \int\int_D 1\,dA$. If $f \geq 0$ then the volume of region under graph is $Vol(E) = \int\int_D f(x,y)\,dA$. Also used to find the area of parametric and graph surfaces.

$\int_C f(x,y,z)\,ds$. Line integral over a curve with repset ot arc length. There are also $\int_C f(x,y,z)\,dx$, and also with dy and dz. These latter are also written $\int_C P\,dx + Q\,dy + R\,dz$. Mostly computed directly or also via Green's theorem. There is also $\int_{-C} = -\int_C$. The parametrization gives the orientation on C. Sometimes the curve C is given without a parametrization so that you may have to find one and you must be careful to check that the resulting orientation is the one that the problem may be asking you.

$\int_C \mathbf{F} \cdot d\mathbf{r}$. Line integral of a vector field along C. It boils down to the one above.

$\int\int_S f(x,y,z)\,dS$. Surface integral. Mostly computed by reducing it to a double integral over the region for the parameters of S. Also computed using Green-Stokes-Gauss theorems.

$\int\int_S \mathbf{F} \cdot d\mathbf{S}$. Surface integral of a vector field (flux). It boils down to the one above. Exercise caution in selecting the appropriate orientation for S.

$\int\int\int_E f(x,y,z)\,dV$. Triple integral over a solid region of space. Mostly computed using Fubini's theorem on iterated integrals and type 1, 2, 3 possibly using also polar at some point. $Vol(E) = \int\int\int_E 1\,dV$.