MAT 205 SPRING 2000
MIDTERM II

Name:

SSN:

THERE ARE FOUR PROBLEMS PLUS ONE EXTRA CREDIT PROBLEM
SHOW YOUR WORK!!!

1. Suppose

$$z = 2x^2y + xe^y$$
$$x(u, v) = v \sin u$$
$$y(u, v) = uv$$

Compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

2. A box has dimensions l, w, and h which vary in the following way:

$$l(t) = t^2 + 2$$
$$h(t) = e^{2t} + 1$$
$$w(t) = 4t^4 + 2t^2 + 2$$

Let $V = V(l, w, h) = l \times w \times h$ be the volume function of the box. Find the rate of change with respect to time of the volume of the box at $t = 0$ seconds.

3. The region D in $\mathbb{R}^2$ shown on the blackboard is bounded by $x = 1$, $y = e^x$, and $y = 1 - x^2$. Compute

$$\int \int_D x dA$$

4. Let $z = f(x, y) = xy^3 - x^2y$.
a. Find the gradient of f at the point $(x, y) = (3, 2)$.

b. Find the equation of the tangent plane to the graph of the surface at the point $(3, 2, 6)$.

c. Let $\mathbf{u}$ be the unit vector pointing in the direction given by $\theta = -\frac{\pi}{3}$. Find the derivative of f in the direction $\mathbf{u}$ at the point $(x, y) = (3, 2)$.

Extra Credit:
Let $f(x, y)$ be a differentiable function on $\mathbb{R}^2$. Two unit vectors $\mathbf{u}$ and $\mathbf{v}$ are given by

$$\mathbf{u} = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$$
$$\mathbf{v} = \frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$$

Assume that $D_{\mathbf{u}}f(0,0) = \frac{3\sqrt{2}}{2}$ and $D_{\mathbf{v}}f(0,0) = \frac{\sqrt{2}}{2}$. Find $\nabla f(0,0)$.