MAT 141 FALL 2002
MIDTERM II

!!! WRITE YOUR NAME, SUNY ID N. AND SECTION BELOW !!!

NAME :

SUNY ID N. :

SECTION :

THERE ARE 6 PROBLEMS. THEY DO NOT HAVE EQUAL VALUE.
SHOW YOUR WORK!!!

1	40	
2	30	
3	30	
4	40	
5	30	
6	30	
Total	200	

1. [**40 points**] Differentiate:

a) $y = \sin x^2 \cdot \cos^2 x$

$$y' = (\sin x^2)' \cdot \cos^2 x + \sin x^2 \cdot (\cos^2 x)'$$
$$= (2x \cos x^2) \cdot \cos^2 x + \sin x^2 \cdot (2 \cos x \sin x)$$

b) $y = \ln (\sin (e^x))$

$$y' = \ln' (\sin (e^x)) \cdot (\sin (e^x))' = \frac{1}{\sin (e^x)} \cdot (\cos (e^x)) \cdot (e^x)'$$
$$= \frac{\cos (e^x)}{\sin (e^x)} e^x$$

c) $y = x^{\sin x}$

Method 1.

$$\ln y = \sin x \cdot \ln x$$
$$(\ln y)' = (\ln x \cdot \sin x)'$$
$$\frac{y'}{y} = (\tfrac{1}{x} \sin x + \ln x \cos x)$$
$$y' = y(\tfrac{1}{x} \sin x + \ln x \cos x) = x^{\sin x}(\tfrac{1}{x} \sin x + \ln x \cos x)$$

Method 2.

$$y = e^{\ln x \cdot \sin x}$$
$$y' = e^{\ln x \cdot \sin x}(\ln x \cdot \sin x)' = e^{\ln x \cdot \sin x}(\tfrac{1}{x} \sin x + \ln x \cos x)$$
$$= y(\tfrac{1}{x} \sin x + \ln x \cos x) = x^{\sin x}(\tfrac{1}{x} \sin x + \ln x \cos x)$$

2. **[30 points]** Find an equation for the line that is tangent to the graph of $y = x^2 \ln x$ and goes through the origin.

$y' = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x$

Method 1.
Pick a point on the curve, say $(a, a^2 \ln a)$. Then the tangent line is,
$y - a^2 \ln a = (2a \ln a + a)(x - a)$
$y = (2a \ln a + a)x - 2a^2 \ln a - a^2 + a^2 \ln a$
$y = (2a \ln a + a)x - a^2 \ln a - a^2$
Since this line passes $(0, 0)$,
$0 = -a^2 \ln a - a^2$
$a^2(\ln a + 1) = 0$
So either $a = 0$ or $\ln a + 1 = 0$. But, we have that $a > 0$, so $\ln a + 1 = 0$.
Hence $\ln a = -1$.
We find that $a = e^{-1}$.
Plugging the value of a back into the equation for the tangent line, we get, $y = (2\frac{1}{e} \cdot (-1) + \frac{1}{e})x + 0$
$y = -\frac{1}{e}x$

Method 2.
Suppose the line passed the point $(a, a^2 \ln a)$ of the curve. Then the slope of the line is,
$\frac{\Delta y}{\Delta x} = \frac{a^2 \ln a}{a} = a \ln a$.
Since this line is also tangent to the graph at $(a, a^2 \ln a)$, we have
$2a \ln a + a = a \ln a$
Solving this, we get $a = \frac{1}{e}$. So the slope is $-\frac{1}{e}$.
The line that passes $(0, 0)$ and has slope of $-\frac{1}{e}$ is
$y = -\frac{1}{e}x$.

3. **[30 points]** Let $f(x) = e^{\sin x}$ for x in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Compute

a) $\frac{df}{dx}|_{x=0}$

$\frac{df}{dx} = \cos x e^{sinx}$ $\frac{df}{dx}|_{x=0} = cos0e^{sin0} = 1 \cdot e^0 = 1$

b) $\frac{df^{-1}}{dx}|_{x=1}$

Since f(0)=1,

$\frac{df^{-1}}{dx}|_{x=1} = \frac{df^{-1}}{dx}|_{x=f(0)} = \frac{1}{\frac{df}{dx}|_{x=0}} = 1$

4. [**40 points**] Sketch the graph of

$$y = -x^3 + 6x^2 - 9x + 3$$

(including: asymptotes if any, absolute and local extrema, if any intervals of increasing and decreasing, concavity).

There are no asymptotes.

$y' = -3x^2 + 12x - 9 = -3(x^2 - 4x + 3) = -3(x-1)(x-3)$
$y'' = -6x + 12$

So we investigate at points $x = 1, 2, 3$.
$y(1) = -1 + 6 - 9 + 3 = -1$, $y'(1) = 0$, $y''(1) = 6$

$y(2) = -8 + 24 - 18 + 3 = 1$, $y''(2) = 0$

$y(3) = -27 + 54 - 27 + 3 = 3$, $y'(3) = 0$, $y''(3) = -6$
So we see that $(1, -1)$ is local min, $(2, 1)$ is inflection point, and $(3, 3)$ is local maximum.

Also, drawing a table,

	x<1	1<x<2	2<x<3	3<x
y'	$-$	$+$	$+$	$-$
y''	$+$	$+$	$-$	$-$
y	$\smile$		$\frown$	

The graph should include all the values of the critical points and inflection points.

5. [**30 points**] Find the linearization of

$$f(x) = e^{2x} - \sin x + 2$$

at $x = 0$ and use it to find an approximate value for $f(-0.1)$.

$f'(x) = 2e^{2x} - \cos x$
$f'(0) = 2 - 1 = 1$
$L(x) = f(0) + f'(0)(x - 0) = 3 + 1(x - 0) = x + 3$
$f(-0.1) \simeq L(-0.1) = -0.1 + 3 = 2.9$

6. [**30 points**] The height of an object moving vertically is given by

$$h(t) = -8t^2 + 48t + 56$$

where h is in feet and t is in seconds. Find:

a) the object's velocity when $t = 0$;

$h' = -16t + 48$
at $t = 0$, $h' = 48$ (feet/sec).

b) its maximum height and when it occurs;

solving $h' = 0$,
$-16t + 48 = 0$
$t = 3$
At $t = 3$, $h(3) = -72 + 144 + 56 = 128$. Since $h'' = -8$ this is a local maximum. The domain of h is $[0, \infty)$.
$h(0) = 56$, $\lim h_{t \to \infty} = -\infty$.
So we see that this is a global maximum. Hence the maximum height is:
128 (feet)

c) its velocity when $h = 0$.

$h = 0$
$-8t^2 + 48t + 56 = 0$
$t^2 - 6t - 7 = 0$
$(t - 7)(t + 1) = 0$
So $t = -1$ or $t = 7$, but time is positive so $t = 7$ (sec).