MAT 141 FALL 2002
MIDTERM I

!!! WRITE YOUR NAME, SSN AND SECTION BELOW !!!

NAME : SSN :

SECTION :

THERE ARE 5 PROBLEMS. THEY DO NOT HAVE EQUAL VALUE.
SHOW YOUR WORK!!!

1	30	
2	50	
3	40	
4	40	
5	40	
Total	200	

1. [**30 points**] Let $f(x) =< x >$ be the decimal part function.
For example: if $x = 2.65$, then $< x >= .65$; if $x = 3.567$, then $< x >= .567$.
Determine the following limit.

$$\lim_{x\to\infty} \frac{< x >}{x}.$$

Solution :
$0 \leq < x > < 1$
$0 \leq \frac{<x>}{x} < \frac{1}{x}$
By the Sandwich Theorem, $\lim_{x\to\infty} 0 \leq \lim_{x\to\infty} \frac{<x>}{x} \leq \lim_{x\to\infty} \frac{1}{x}$

$0 \leq \lim_{x\to\infty} \frac{<x>}{x} \leq 0$

Answer: $\lim_{x\to\infty} \frac{<x>}{x} = 0$

2. **[50 points]** Let

$$f(x) = x^3\left(\frac{1}{x-1} - \frac{1}{x+1}\right)$$

Find the asymptotes of $f(x)$ and draw them on the xy-plane together with a qualitative sketch of the graph of the curve near the asymptotes.

Solution)

Vertical Asymptotes: $x - 1 = 0$ and $x + 1 = 0$.
$f(x) = \frac{2x^3}{x^2-1} = \frac{2x^3-2x+2x}{x^2-1} = 2x + \frac{2x}{x^2-1}$

Oblique Asymptotes: $y = 2x$

Horizontal Asymptotes: None, since

$$\lim_{x\to\infty} \frac{x^3}{x^2-1} = \infty$$

and

$$\lim_{x\to-\infty} \frac{x^3}{x^2-1} = -\infty$$

3. [**40 points**] Let $f(x)$ be a function defined for every value of x.
Assume that
a) $f(x)$ is differentiable for every value of x;
b) for $x \neq 0$, $f(x) = \frac{\sin(x^3+x^2)}{\sqrt{x^3+x^2}}$.
Find $f(0)$. Justify your answer.

Solution)
Since f(x) is differentiable, it is continuous. So $\lim_{x\to 0} f(x) = f(0)$ holds.
Letting $t = x^3 + x^2$,

$$\lim_{x\to 0} \frac{\sin(x^3+x^2)}{\sqrt{x^3+x^2}} = \lim_{t\to 0^+} \frac{\sin t}{\sqrt{t}} = \lim_{t\to 0^+} \sqrt{t}\frac{\sin t}{t} = 0 \cdot 1 = 0$$

Hence, $f(0) = 0$.

4. [**40 points**] Let u and v be differentiable functions.
Assume that

$$u(1) = 1, \quad u'(1) = -1, \quad v(1) = 2, \quad v'(1) = -3.$$

Compute:

a) $(\frac{u}{v})'(1)$

$(\frac{u}{v})'(1) = \frac{u'v - uv'}{v^2}(1) = \frac{u'(1)v(1) - u(1)v'(1)}{v(1)^2}$
$= \frac{-1\cdot 2 - 1\cdot(-3)}{2^2} = \frac{-2+3}{4} = .25$

b) $(uv^2)'(1)$

$(uv^2)'(1) = \{u' \cdot v^2 + u \cdot (v^2)'\}(1) = \{u' \cdot v^2 + u \cdot (v'v + vv')\}(1)$
$= u'(1)v(1)^2 + u(1)\cdot(v'(1)v(1) + v(1)v'(1)) = -1\cdot 2^2 + 1\cdot(-3\cdot 2 + 2\cdot -3) =$
$-4 - 12 = -16$

5. [**40 points**] A body moves along the s-axis with velocity

$$v = t^2 - 4t + 3.$$

a) Find the body's acceleration each time the velocity is zero.

$v = 0$ when $t^2 - 4t + 3 = 0$. $(t-3)(t-1) = 0$, so $t = 1$ or $t = 3$.
$a = v' = 2t - 4$
So when $t = 1$ acceleration is $a = -2$ and when $t = 3$, $a = 2$.

b) When is the body moving forward?

Body moves forward when $v > 0$. $(t-1)(t-3) > 0$. So $t < 1$ or $t > 3$.

c) When is the body's velocity increasing?

$v' > 0$. So $2t - 4 > 0$.
$t > 2$. After t=2 velocity increases.