Homework 3 (MAT 562)

(1) Let (W, ξ) be a contact manifold of dimension $2n-1$ and let $\alpha \in \Omega^1(W)$ satisfy $\xi = \ker(\alpha)$. Show that $\alpha \wedge d\alpha^{n-1}$ does not vanish at any point in W.

(2) Let (M, ω) be a symplectic manifold of dimension 4 or higher. Let X be any vector field satisfying $\mathcal{L}_X \omega = f\omega$ for some nowhere vanishing smooth function

$$f : M \longrightarrow \mathbb{R} - \{0\}.$$

Show that f must be a constant function.

(3) Let E be an oriented vector bundle with a conformal symplectic structure. Show that E admits a symplectic structure.

(4) (McDuff-Salamon). Let (M, ω) be a symplectic manifold of dimension ≥ 4 and let $W \subset M$ be a compact hypersurface. Let X, X' be a Liouville vector fields transverse to W. Show that both vector fields give the same coorientation for W (I.e. they induce isotopic trivializations of the normal bundle of W).
Hint: Consider $X - X'$.

(5) Show that the Thurston-Bennequin number of a Legendrian knot can be computed from its Legendrian front diagram in terms of the writhe and the number of cusps.