Homework 2 (MAT 562)

(1) Suppose that M is a smooth manifold with symplectic forms ω and ω'. Let $K \subset M$ be a closed subset so that ω and ω' agree on $T_x M$ for each $x \in K$. Show that there is a diffeomorphism $\phi : M \longrightarrow M$ so that $\phi(x) = x$ for each $x \in K$ and so that ω and $\phi^* \omega'$ agree on a neighborhood of K.

I made a mistake with this question. The closed subset K needs an additional property, such as being a smooth deformation retraction from an open neighborhood. I am sorry about this.

(2) Let (M, ω) and (M', ω') be two symplectic manifolds with coisotropic submanifolds $C \subset M$ and $C' \subset M'$. Let $f : C \longrightarrow C'$ be a diffeomorphism so that $f^*(\omega'|_{C'}) = \omega|_C$. Show that there are neighborhoods N of C and N' of C' and a symplectomorphism $\phi : (N, \omega) \longrightarrow (N', \omega')$ satisfying $\phi(x) = f(x)$ for each $x \in C$.

(3) Classify all closed 2-dimensional symplectic manifolds up to symplectomorphism (you may assume the classification of surfaces).

(4) Let (M, ω) be a symplectic manifold. Show that the Hamiltonian symplectomorphism group $Ham(M, \omega)$ acts transitively on M.

(5) Let (M, ω) be a symplectic manifold and $\iota_t : L \hookrightarrow M$, $t \in [0, 1]$ be a smooth family of Lagrangian embeddings. Show that there is a neighborhood $U \subset T^*L$ of the zero section and smooth family of symplectic embeddings $\widetilde{\iota}_t : U \hookrightarrow M$, $t \in [0, 1]$ satisfying $\widetilde{\iota}_t(x) = \iota_t(x)$ for each $x \in L$.