MAT 542 Homework 2

(1) (Hatcher 4.2 1): Use homotopy groups to show that there is no retraction $r : \mathbb{R}P^n \longrightarrow \mathbb{R}P^k$ for $n > k > 0$ (I.e. a continuous map $r : \mathbb{R}P^n \longrightarrow \mathbb{R}P^k$ satisfying $r|_{\mathbb{R}P^k} = \mathrm{id}$).
(2) (Hatcher 4.2 8) Show that the suspension of an acyclic CW complex is contractible. (Recall that a CW complex is *acyclic* if all reduced homology groups vanish.)
(3) Compute $\pi_3(SU(n))$ for each n.
(4) (Hatcher 4.2 22): Show that $H_{n+1}(K(G,n);\mathbb{Z}) = 0$. (Hint: construct a $K(G,n)$ space with an explicit $n+1$ skeleton).
(5) Let $p : E \longrightarrow B$ be a Serre fibration whose fibers are contractible and so that B is a CW complex. Show that p has a continuous section (I.e. Show that there is a continuous map $s : B \longrightarrow E$ so that ps is the identity map).