Homework 4

(1) Use obstruction theory to show that every closed smooth manifold with vanishing Euler characteristic has a nowhere vanishing vector field. You may use the fact that the Euler characteristic is the degree of the Euler class.

(2) Show that the Euler characteristic of the boundary of an odd dimensional smooth manifold is even (Hint: construct a closed manifold by gluing the odd dimensional smooth manifold along its boundary and use the fact that its Euler characteristic vanishes by Poincaré duality). Use this fact to compute the unoriented cobordism group in dimension 2.

(3) Compute the framed cobordism groups in dimension 1. Use this to compute $\pi_4(S^2)$.

(4) Compute all Steenrod squares of $\mathbb{CP}^n$.

(5) Compute $H^*_{\mathbb{Z}/2\mathbb{Z}}(S^1; \mathbb{Z}/2\mathbb{Z})$ as a $\mathbb{Z}/2\mathbb{Z}[t]$-module where $\mathbb{Z}/2\mathbb{Z}$ acts on S^1 via a reflection map.