

HOMEWORK 1

- (1) Let $(h_k)_{k \in \mathbb{Z}}, (\partial_k)_{k \in \mathbb{Z}}$ be a generalized homology theory (i.e. satisfying the Eilenberg-Steenrod axioms minus the dimension axiom). Show that there is a Mayer-Vietoris sequence for this generalized homology theory.
- (2) Compute $H_{\mathbb{Z}/2\mathbb{Z}}^*(S^1; \mathbb{Z}/2\mathbb{Z})$ as a $\mathbb{Z}/2\mathbb{Z}[t]$ -module where $\mathbb{Z}/2\mathbb{Z}$ acts on S^1 via a reflection map.
- (3) Compute all the Steenrod squares of $\mathbb{C}P^n$.
- (4) Let M be a closed connected $2n$ -manifold satisfying:
 - $H_i(M; \mathbb{Z}/2\mathbb{Z}) = 0$ for $0 < i < n$ and
 - $H_n(M; \mathbb{Z}/2\mathbb{Z}) = \mathbb{Z}/2\mathbb{Z}$.Prove that n is a power of 2.