

## HOMEWORK 2

**Due:** Thursday September 13th at 10:00am in Physics P-124

*Please write your solutions legibly; the TA may disregard solutions that are not readily readable. All solutions must be stapled (no paper clips) and have your name (first name first) and HW number in the upper-right corner of the first page.*

**Problem 1:** Let  $E, F$  be measurable sets in  $\mathbb{R}$ . Show

$$m(E \cup F) = m(E) + m(F) - m(E \cap F).$$

**Problem 2:** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be continuous. Show that  $f^{-1}(I)$  is measurable for any interval  $I$ .

**Problem 3:** Show that  $E \subset \mathbb{R}$  is measurable iff

$$m^*(I) = m^*(E \cap I) + m^*(I - E \cap I)$$

for every interval  $I$ .

**Problem 4: (Optional).** Let  $A \subset \mathbb{R}$  be a subset satisfying the following two properties:

- $a - b \notin \mathbb{Q} - \{0\}$  for all  $a, b \in A$  and
- for each  $x \in \mathbb{R}$  there exists  $a \in A$  so that  $x - a \in \mathbb{Q}$ .

Such a set exists by the axiom of choice. Show that  $A$  is not measurable.