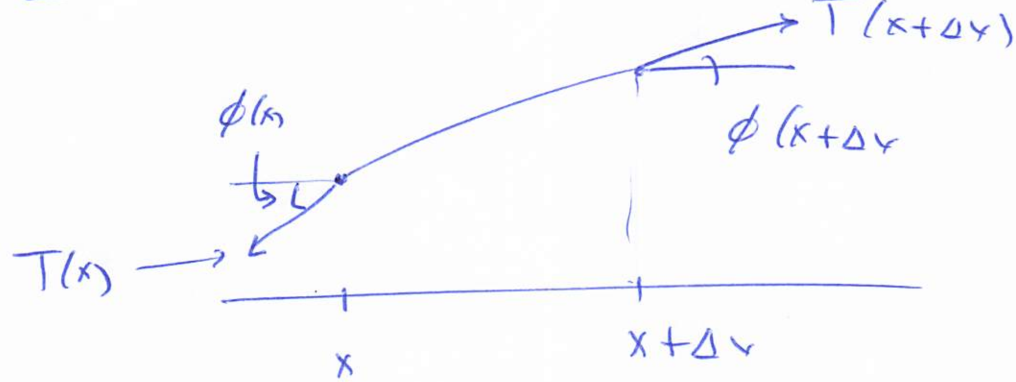
VI a 3.1 3.2

§3.1 Vibrating String



horizontal Force:

$$-T(x)\cos\phi(x) + T(x+\Delta x)\cos\phi(x+\Delta x) = 0$$

So $T(x)\cos\phi(x) \equiv H$

Vertical Force: $F = m \cdot a$.

$$-T(x)\sin\phi(x) + T(x+\Delta x)\sin\phi(x+\Delta x) = m\frac{\partial^2 u}{\partial t^2}$$

accelaration

$$T(x) = \frac{H}{\cos\phi(x)}$$

$$\rho\,\Delta x \frac{\partial^2 u}{\partial t^2}(x,t) = -H\tan\phi(x) + H\tan\phi(x+\Delta x)$$

$$\tan\phi(x) = \frac{\partial u}{\partial x}$$

$$\rho\,\Delta x\,\frac{\partial^2 u}{\partial t^2} = -H\frac{\partial u}{\partial x}(x,t) + H\frac{\partial u}{\partial x}(x+\Delta x, t)$$

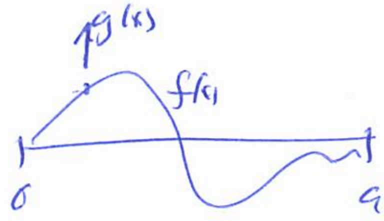
$$= H\frac{\partial^2 u}{\partial x^2}(x,t)\,\Delta x$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = \frac{H}{\rho}\frac{\partial^2 u}{\partial x^2}} \qquad 0 < x < a,\ t > 0$$

$$H/\rho = \frac{1}{c^2} \qquad (c\ \ m/s)$$

§3.2

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2} \\ u(0,t) = 0 \\ u(a,t) = 0 \\ u(x,0) = f(x) & \text{initial shape} \\ \dfrac{\partial u}{\partial t}(x,0) = g(x) & \text{initial velocity.} \end{cases}$$



Find Solution

I) The problem is linear. (No need for steady state)
We can add sol.

II) Sep. of Variables.

$$u(x,t) = \phi(x)T(t)$$

$$\frac{\phi''(x)}{\phi(x)} = p = \frac{1}{c^2}\frac{T''(t)}{T(t)}$$

$p = -\lambda^2$ $\quad$ ($p=0$, $p>0$ no usefull sol.)

$$\begin{cases} \phi''(x) + \lambda^2 \phi(x) = 0 \\ T''(t) + c^2\lambda^2 T(t) = 0 \\ \phi(0) = 0 \quad \phi(a) = 0 \end{cases}$$

$$\phi_n(x) = \sin \lambda_n x \qquad \lambda_n^2 = \left(\frac{n\pi}{a}\right)^2$$

$$T_n(t) = a_n \cos \lambda_n t + b_n \sin \lambda_n t.$$

$$u_n(x,t) = \sin \lambda_n x \left[a_n \cos \lambda_n c t + b_n \sin \lambda_n c t \right]$$

$$\boxed{u(x,t) = \sum_{n \geq 1} \sin \lambda_n x \left[a_n \cos \lambda_n c t + b_n \sin \lambda_n c t \right]}$$

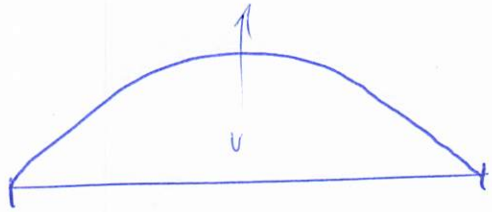
III Initial data

$$\begin{cases} u(x,0) = f(x) \\ \dfrac{\partial u}{\partial t}(x,0) = g(x) \end{cases}$$

$$\begin{cases} u(x,0) = \sum_{n\geq 1} \sin\lambda_n x \cdot a_n = f(x) \\ \frac{\partial u}{\partial t}(x,0) = \sum_{n\geq 1} b_n \cancel{\frac{n\pi}{a}} \lambda_n c \sin\lambda_n x = g(x) \end{cases}$$

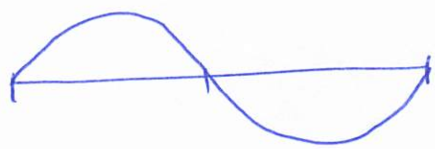
$$\begin{cases} a_n = \frac{2}{a}\int_0^a f(x) \sin\lambda_n x \, dx \\ b_n = \frac{2}{n\pi c}\int_0^a g(x) \sin\lambda_n x \, dx \end{cases}$$

$$\lambda_n = \frac{n\pi}{a}$$



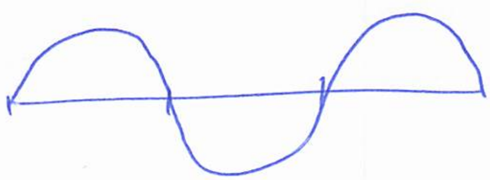
$n=1$

$$\boxed{u_n(x,t) = \sin\lambda_n t \cdot \cos\lambda_n c t}$$

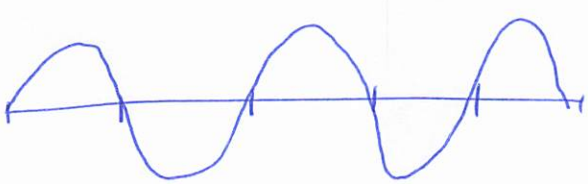


$n=2$

$n=3$



$n=5$



The time to return to starting shape:

$$\cos \lambda_n c t = 1$$

$$\lambda_n c t = 2\pi \qquad T_n = \frac{2\pi}{\lambda_n c} = \frac{2a}{nc}$$

Number of cycles per sec $= 1/T_n$.

$$\boxed{\text{frequency} = \frac{nc}{2a}}$$

$$\frac{c}{2a},\ 2\cdot\frac{c}{2a},\ 3\,\frac{c}{2a},\ \ldots$$

$f_n = \text{freq}_n = n\cdot\left(\frac{c}{2a}\right) = n\cdot f_1$. harmonics

$u(x,t)$ is periodic with period $\frac{1}{f_1} = T_1$

Review Midterm I

Time: In Class

Material: Chapter 0, 1, 2 (everything from the schedule).

2 Problems, like HW.

What to pay attention to?

Chapter 0: . how to solve linear Diff. Eq. with constant coefficients

. how to solve nonhomogeneous linear DE.

Chapter I:

periodic functions

Fourier Series

Even and Odd functions

Fourier Sin and cos - series.

convergence of Fourier Series, Thm p69.

Uniform convergence Thm 1 p.73

§1.3 Thm 2 p.73 Thm 3 p.76

§1.5: Thm 1, 2, 3, 4, 6, 7

§1.9: Fourier Integral Representation Thm.

Chapter II:

§2.1 Derivation Heat Eq.

You have to know the method how to Solve the heat Equation with all types of boundary conditions, see examples in the book.

§2.11 Infinite rod. and the general Solution (7) p. 192.