III a) §1.5)

Thm a_n, b_n Fourier coef of f

$\tilde{a}_n\ \tilde{b}_n$ F ——— $\tilde{f}$

the $ca_n + d\tilde{a}_n$ are ——— of $cf + d\tilde{g}$.

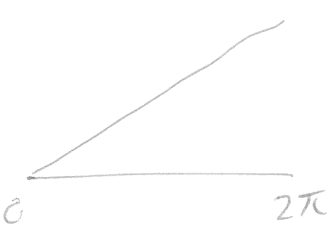
Thm f piecewise ~~smooth~~ C^0

$$\int_a^b f(x)\,dx = \int a_0 + \sum \int (a_n \cos nx + b_n \sin nx)\,dx$$

Thm f, g PC

$$\int_a^b fg = \int_a^b a_0 g + \sum \int (a_n \cos nx + b_n \sin nx)\, g(x)\,dx.$$

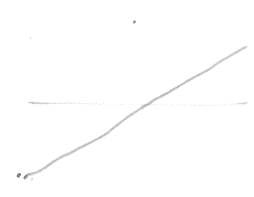
Ex)



$h(x) = x$

$$h(x) = \pi - 2\sum_{n=1}^{\infty} \frac{\sin nx}{n}$$

$$f = \frac{\pi - h(x)}{2} =$$



$$= \sum \frac{\sin nx}{n}$$

$$\int_0^b f(x)\,dx = \sum_{n\geq 1} \frac{1-\cos nb}{n^2}$$

$$\parallel$$

$$\int \frac{\pi - x}{2} = \frac{\pi}{2}b - \frac{b^2}{4}$$

$$\frac{x(2\pi - x)}{4} = \sum_{n\geq 1} \frac{1}{n^2} - \sum \frac{\cos nx}{n^2}$$

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{x(2\pi - x)}{4} = \sum \frac{1}{n^2} - \sum \int_0^{2\pi} \cos nx = \sum \frac{1}{n^2}$$

$$\parallel$$

$$\frac{\pi^2}{6} \qquad \boxed{\sum \frac{1}{n^2} = \frac{\pi^2}{6}}$$

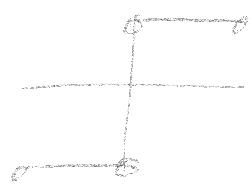
<u>Thm</u> f periodic C^0 piecewise smooth the differentiated Fourier series of f is ~~the~~ converges to $f'(x)$ $\forall x$ $f''(x)$ exists.

$$f'(x) = \sum_{n\geq 1} -n a_n \sin nx + n b_n \cos nx$$

Ex) $f(x) = |x| \qquad -\pi < x < \pi$

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\cos x + \frac{\cos 3x}{9} + \frac{\cos 5x}{25} + \cdots \right)$$

$$f'(x) = \frac{4}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots \right)$$

$f' =$ 

no f''

Thm: f periodic

If $\sum |n^k a_n| + |n^k b_n| < \infty$

then $f, f^{(1)}, \ldots, f^{(k)}$ continuous.

with differentiated Fourier Series

Ex) $f(x) = \sum e^{-n\alpha} \cos nx \qquad \alpha > 0$

f is C^∞

§1.9

f defined on $(-\infty, \infty)$ and PS on each finite interval.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{a} + b_n \sin\frac{n\pi x}{a}$$

$$a_0 = \frac{1}{2a}\int_{-a}^{a} f(x)\,dx$$

$$a_n = \frac{1}{a}\int_{-a}^{a} f(x)\cos\frac{n\pi x}{a} \qquad b_n = \frac{1}{a}\int_{-a}^{a} f(x)\sin\frac{n\pi x}{a}\,dx$$

Now let $\lambda_n = \frac{n\pi}{a}$

$$A_a(\lambda_n) = \frac{1}{\pi}\int_{-a}^{a} f(x)\cos\lambda_n x$$

$$B_a(\lambda_n) = \frac{1}{\pi}\int_{-a}^{a} f(x)\sin\lambda_n x$$

So $\quad a_n = \frac{\pi}{a} A_a(\lambda_n) \qquad b_n = \frac{\pi}{a} B_a(\lambda_n)$

So

$$f(x) = a_0 + \sum \left[A_a(\lambda_n) \cos \lambda_n x + B_a(\lambda_n) \sin \lambda_n x \right] \Delta\lambda$$

$$\Delta\lambda = \frac{\pi}{a} = \lambda_{n+1} - \lambda_n$$

$$\boxed{\begin{aligned} A_a(\lambda) &\xrightarrow[a\to\infty]{} \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos(\lambda x)\,dx = A(\lambda) \\ B_a(\lambda) &\longrightarrow \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\sin(\lambda x)\,dx = B(\lambda) \end{aligned}}$$

$a_0 \longrightarrow 0$

$$f(x) = \int_0^{\infty} \left[A(\lambda)\cos(\lambda x) + B(\lambda)\sin \lambda x \right] d\lambda$$

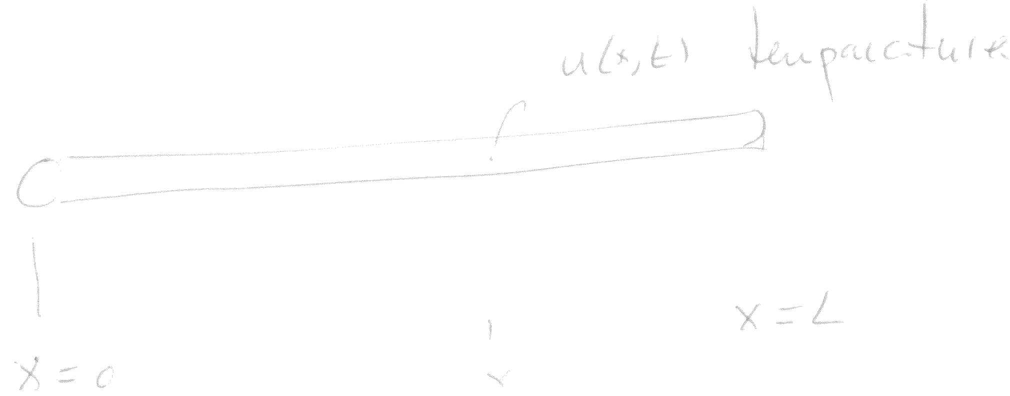
Thm f PS on every finite interval

$\int |f| < \infty$ Then

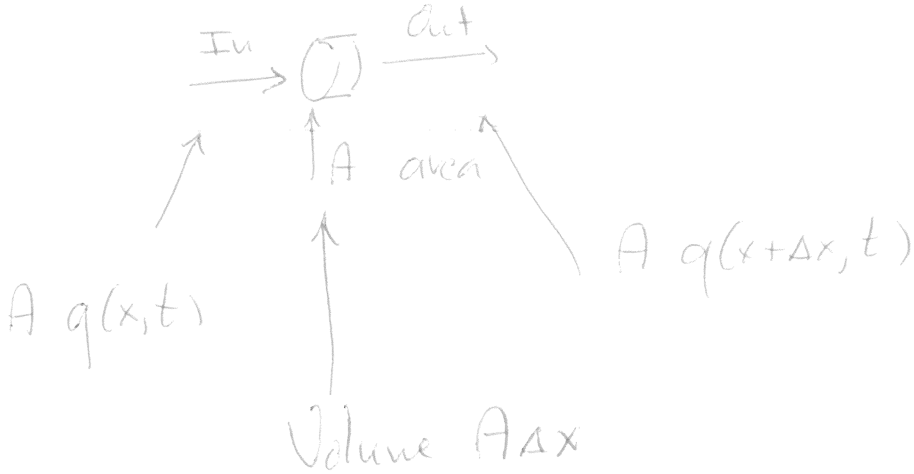
$\frac{1}{2}(f(x_+) + f(x_-)) =$

Fourier $\int$ representation.

III b) § 2.1) ??



Heat flux $q(x,t)$ $\longrightarrow +$



Heat in $\square$ = $\rho c \cdot$ vol. $u(x,t)$

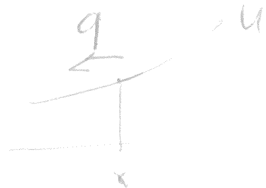
$$\rho c\, A \Delta x \frac{\partial u}{\partial t} = A q(x,t) - A q(x+\Delta x, t) + A\Delta x\, g$$

internal generation

$$\rho c \frac{\partial u}{\partial t} = -\frac{\partial q}{\partial x} + g$$

Fourier law of heat conduction

$$\boxed{q = -\kappa \frac{\partial u}{\partial x}}$$



$$\boxed{\rho c \frac{\partial u}{\partial t} = \kappa \frac{\partial^2 u}{\partial x^2} + g}$$

No generation

$$\boxed{\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}} \qquad k = \frac{\kappa}{\rho c}$$

$0 < x < a$

$0 < t$

Many solution $u = x^2 + 2kt \qquad e^{-kt}\sin x$

Initial Condition: $u(x,0) = f(x)$

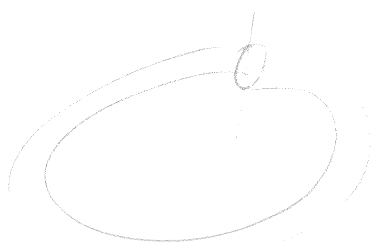
boundary Condition: $\left.\begin{cases} u(0,t) = T_0 \\ u(a,t) = T_1 \end{cases}\right\}$ <u>Dirichlet</u>

or $\qquad u(x_0,t) = \alpha(t)$

a $\quad \frac{\partial u}{\partial x}(x_0,t)=\beta(t)$ Neuman (2nd type)

fix flux at some point.

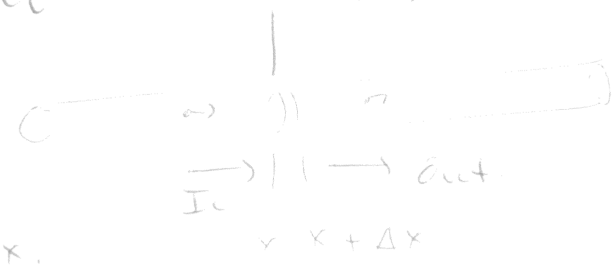
Insulated place $\frac{\partial u}{\partial x}(x_0,t)\equiv 0$



$$\begin{cases} \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(a,t) \\ u(0,t)=u(a,t) \end{cases}$$

Initial Value-Boundary Problem

Diffusion $\quad u(x,t)$ concentration.



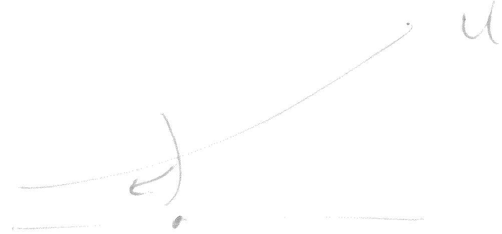
q flux.

$$\frac{\partial u}{\partial t}\cdot Vol = q(x,t) - q(x+\Delta x,t)$$

Vol $= A\Delta x$.

$$k\frac{\partial u}{\partial t} = -\frac{\partial q}{\partial x}$$

<u>Diffusion</u>:



$$q = -c\frac{\partial u}{\partial x}$$

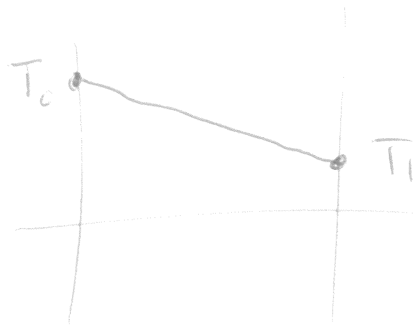
$$\boxed{\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}}$$ Diffusion Equation.

§7.2| <u>Steady State</u>

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t} & 0 < x < a \\ u(0,t) = T_0 & \\ u(a,t) = T_1 & \\ u(x,0) = f(x) & \end{cases}$$

It stabilizes $\begin{cases} u(x,t) \xrightarrow{t} v(x) \\ \dfrac{\partial u}{\partial t} \to 0 \end{cases}$

$$\begin{cases} \dfrac{d^2v}{dx^2} = 0 \\ v(0) = T_0 \\ v(a) = T_1 \end{cases}$$



$$v(x) = Ax + B$$

$$\boxed{v(x) = T_0 + (T_1 - T_0)\frac{x}{a}}$$

Ex)
$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t} \\ \dfrac{\partial u}{\partial x}(0,t) = 0 \\ \dfrac{\partial u}{\partial x}(a,t) = 0 \\ u(x,0) = f(x) \end{cases}$$

Steady State?

$$\frac{d^2v}{dx^2} = 0 \quad \frac{dv}{dx}(0) = \frac{dv}{dx}(a) = 0$$

$$v(x) \equiv T$$

transient

$u(x,t)$ solution of IBV-proble

$v(x)$ steady state sol.

$$u(x,t) = w(x,t) + v(x)$$

transient Steady State

Ex) $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u}{\partial t}$

$u(0,t) = T_0 \qquad u(a,t) = T_1$

$u(x,0) = f(x)$

$$\begin{cases} \dfrac{\partial^2 w}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial w}{\partial t} \\ w(0,t) = 0 \qquad w(a,t) = 0 \\ w(x,0) = f(x) - v(x) \end{cases}$$

$v(x) = T_0 + \dfrac{T_1 - T_0}{a} x,$

homogeneous

$\begin{cases} \text{B-Cond and} \\ \text{Diff. Eq.} \end{cases}$