Ia

0.1, 0.2, 0.3, 0.4

Frey 105   9:30 – 10:50

Off. Hrs TuTh 11.30 – 13:00

HW 15% weekly due in Tu Class

MI, MII, 25%

F   35 %

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↳ Teaching

↳ MAT341

Syllabus

Schedule

Mat

HW

0.1 ODE

A) 1^{st} order

$$\begin{cases} \dfrac{du}{dt} = k(t)\,u \\ u(t_0) = u_0 \end{cases}$$

Thm: $\exists!$ sol.

Technique to find sol.

1) $\dfrac{1}{u}\dfrac{du}{dt} = k(t)$

2) $\ln|u| = \int k + C$

3) $u = Ce^{\int k}$

Ex: $\dfrac{du}{dt} = -tu \implies u = ce^{-\frac{1}{2}t^2}$.

$\dfrac{du}{dt} = ku \implies u = Ce^{kt}$

Obs: u_1, u_2 sol $\implies a_1u_1 + a_2u_2$ Sol.

B <u>2^{nd} order</u>

$$\frac{d^2u}{dt^2} + k(t)\frac{du}{dt} + p(t)\,u = 0$$

<u>Thm</u> u_1, u_2 sol $\Longrightarrow$ $c_1u_1 + c_2u_2$ sol.

2^{nd} order: $\exists\ u_1, u_2$

every sol is a linear comb.

There are diff. techniques to find two independent sol.

1) Constant coef.

Try $u = e^{mt}$

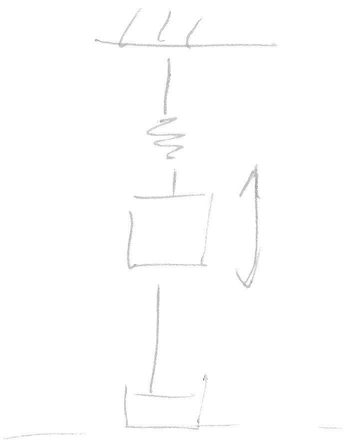
$$m^2 + km + p = 0$$

$$m = \alpha \pm i\beta$$

$$u = e^{\alpha t} e^{i\beta t}$$

Ex: $u'' - \lambda^2 u = 0$

$$u = c_1 e^{\lambda t} + c_2 e^{-\lambda t}$$



friction force spring

$$\begin{cases} u'' + bu' + \omega^2 u = 0 \\ u(0) = u_0 \qquad u'(0) = u_0' \end{cases}$$

$b=0$: $\quad u = c_1 \cos\omega t + c_2 \sin\omega t$.

ossilation

$0 < \frac{b}{2} < \omega$: $\quad u = e^{-\frac{1}{2}bt}(c_1 \cos\omega t + c_2 \sin\omega t)$.

$\frac{b}{2} = \omega$: $\quad u = e^{-\frac{1}{2}bt}(c_1 + c_2 t)$

$\frac{b}{2} > \omega$: $\quad u = c_1 e^{m_1 t} + c_2 e^{m_2 t}$

Other examples

- $t^2 u'' + kt u' + pu = 0$

  Try t^m

Second Ind. Sol

$$u'' + k(t)u' + p(t)u = 0$$

u_1 is sol.

Try $\quad u = v \cdot u_1$

$$u_1 v'' + (2u_1' + k(t)u_1)v' = 0$$

1^{st} order in v'

Higher order lin eq: Same facts

0.2 | Non homogeneous Eq

$$u' + k(t)u = f(t)$$

$$u'' + k(t)u' + p(t)u = f(t)$$

Thm general sol. of linear

$$u = u_c(t) + \underbrace{u_p(t)}_{\text{any particular}}$$

A undetermined coef

Ex 1) $u'' - u = te^{-t}$

Gus: $u_c = c_1 e^{t} + c_2 e^{-t}$

Guess $u_p = (A_0 + A_1 t) e^{-t}$

Ex 2) undamped forced $\nu \neq \omega$

$$u'' + \omega^2 u = f_0 \cos \nu t$$

$u_{gen} = c_1 \cos \omega t + c_2 \sin \omega t$

Try $u_p = A \cos \nu t$

$$u_p = \frac{f_0}{\omega^2 - \nu^2} \cos \nu t$$

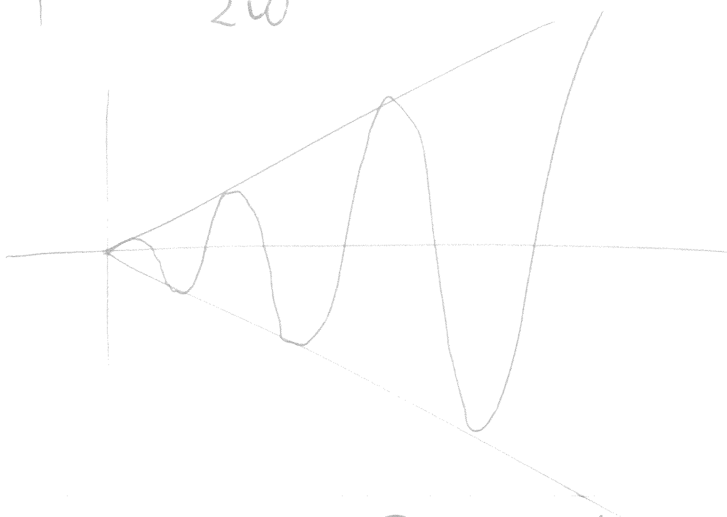
$$u = c_1 \cos \omega t + c_2 \sin \omega t + \frac{f_0}{\omega^2 - \nu^2} \cos \nu t$$

$\nu \neq \omega$.

resonance $\nu = \omega$

$$u_p = At\cos\omega t + Bt\sin\omega t.$$

$$u_p = \frac{f_0}{2\omega} t\sin\omega t$$



(B) Variation of Parameters

$$u' = k(t)u \qquad u_c$$

$$u' = k(t)u + f.$$

Try $u = v \cdot u_c$

$\vdots$

$$u_c \frac{dv}{dt} = f(t)$$

Ex]
$$\frac{du}{dt} - 5u = t$$

$$u_c = Ce^{5t}$$

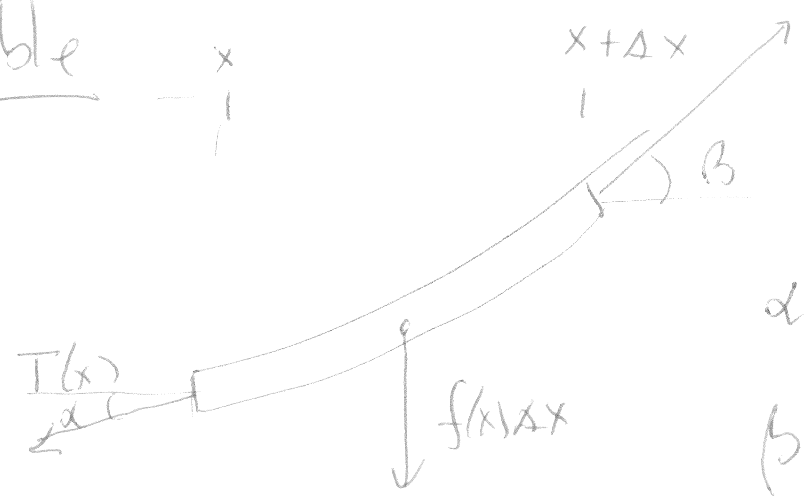
Try $u = v(t)\, e^{5t}$

$$\frac{dv}{dt} = t\, e^{-5t}$$

$$v = \int t\, e^{-5t} = \cdots$$

6.3 Boundary Value Problems

Cable



$\alpha = \phi(x)$ $\cancel{\frac{du}{dx}}$

$\beta = \phi(x + \Delta x)$

hor: $T(x+\Delta x)\cos\phi(x+\Delta x) - T(x)\cos\phi(x) = 0$

ver: $T(x+\Delta x)\sin\phi(x+\Delta x) - T(x)\sin\phi(x) - f(x)\Delta x = 0$

$T(x) \cos\phi(x) = H$ const.

$H \tan\phi(x+\Delta x) - H \tan\phi(x) = f(x)\,\Delta x$

$H \frac{du}{dx}(x+\Delta x) - H \frac{du}{dx}(x) = f(x)$.

$\tan\phi = \frac{du}{dx}$

$$\boxed{H \frac{d^2u}{dx^2} = f(x)}$$

Ex Say $f(x) \equiv w$

$\frac{d^2u}{dx^2} = \frac{w}{H}$ $\quad h(0) = h_0 \quad h(L) = h_1$.

$u = Ax^2 + Bx + C$

$u = \frac{w}{2H}x^2 + Bx + C$

etc.

I b) §1.1

A function is periodic with period p

$$f(x+p)=f(x) \quad \forall x$$

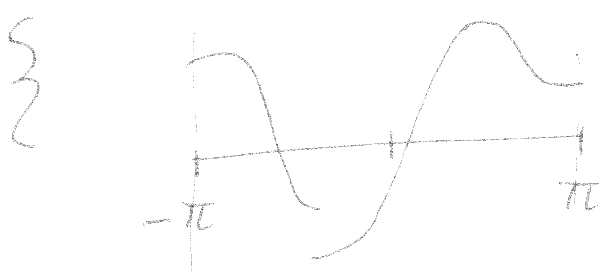
The smallest positive p is <u>the</u> period

$\sin(x)$ $\quad p = 2\pi$

$\cos(x)$ $\quad p = 2\pi$

$\sin(nx)$ $\quad p = \frac{2\pi}{n}$

Example: f period 2π



Rmk: Every function with domain $[-\pi,\pi)$ defines a periodic function of with period 2π.

Ex

$$\frac{d^2u}{dx^2} + u = A\cos 3x$$

$$\hookrightarrow \quad u = c_1\cos x + c_2 \sin x + -\frac{A}{8}\cos 3x$$

Idea

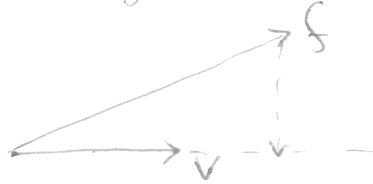
Every periodic function can be expressed as a sum of $\sin nx$ and $\cos nx$.

$$V = \left\{ f : [-\pi,\pi] \longrightarrow \mathbb{R} \;\middle|\; \int_{-\pi}^{\pi} f^2(x)\,dx < \infty \right\}.$$

- V is a vector space.
- Define inner product: $f, g \in V$

$$(f,g) = \frac{1}{\pi}\int_{-\pi}^{\pi} f g \, dx$$

Recall. $f \perp g \iff (f,g) = 0$



$\|v\| = 1$: orthogonal projection has length (f,v)

- $\cos nx, \sin nx \in V$

Lemma $(\sin nx, \cos mx) = 0$

$$(\sin nx, \sin mx) = \begin{cases} 0 & n \neq m \\ 1 & n = m \end{cases}$$

$$(\cos nx, \cos mx) = \begin{cases} 0 & n \neq m \\ 1 & n = m. \end{cases}$$

The functions $\sin nx, \cos nx$ are orthogonal unit vectors

They form a basis!!

Thm $\forall f \in V$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = (f, \cos nx) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$b_n = (f, \sin nx) = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx.$$

Observation

$$f(x) = a_0 + \sum_{n\geq 1} a_n \cos nx + \sum_{n\geq 1} b_n \sin nx$$

$$(f, \cos mx) = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx =$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\left[a_0 \cos mx + \sum_{n\geq 1} a_n \cos nx \cos mx + \sum_{n\geq 1} b_n \sin nx \cos mx\right]$$

$$= 0 \quad a_m \frac{1}{\pi}\int_{-\pi}^{\pi} \cos mx \cos mx \, dx + 0$$

$$= a_m.$$

The coefficients a_n, b_n are the projections of f to the basis vectors

Fourier Coefficients.