Project II MAT312
Group Isomormpisms

Let $G_n \subset \mathbb{Z}_n$ be the set of invertible congruence classes.

Question 1: Show that if $a, b \in G_n$ then

- $ab \in G_n$
- $a^{-1} \in G_n$
- $ab = ba$

Question 2: Construct the multiplication tables for G_3 and G_4.

Question 3: Relabel the elements of G_3 with a and b:

$$1 \mapsto a \text{ and } 2 \mapsto b.$$

Such a relabelling is called an *isomorphism*.
Rewrite the multiplication table of G_3 in terms of a and b.
Same for G_4.
What is the meaning of the results?

Question 4: Construct the multiplication tables for G_5, G_8, G_{10} and G_{12}. Why are they symmetric?

Question 5: Construct the addition table for $\mathbb{Z}_4$.
Relable the elements of $\mathbb{Z}_4$ with a, b, c, d as follows

$$0 \mapsto a \text{ and } 1 \mapsto b \text{ and } 2 \mapsto c \text{ and } 3 \mapsto d$$

and rewrite the addition table for $\mathbb{Z}_4$ in terms of a, b, c, d.

Question 6: Which *Groups* of among G_5, G_8, G_{10} and G_{12} can be relabelled such that their multiplication tables become the same as the addition table for $\mathbb{Z}_4$ (use the letters a, b, c, d)?

Question 7: are there relabellings of G_8 and G_{12} such that their multiplication tables become the same?

Question 8: are there relabellings of G_7 and G_9 such that their multiplication tables become the same?

Question 9: Can you relable G_9 and $\mathbb{Z}_6$ such that the multiplication table of G_9 becomes the same as the addition table for $\mathbb{Z}_6$?

You have two weeks to finish the report. It should be readable by anybody and just a couple of pages. In essence the report should just answer the six questions.