Project I MAT312

Slowest Speed for Euclidean Algorithm

Given a pair of numbers $b > a > 0$ the Euclidean Algorithm determines the greatest common divisor of b and a. See Section 1.1 of the book.

$$\begin{aligned} b &= q_0 a + r_1 \\ a &= q_1 r_1 + r_2 \\ &= \cdots \\ r_{k-1} &= q_k r_k + r_{k+1} \\ &= \cdots \\ r_{n-1} &= q_n r_n + 0 \end{aligned}$$

Which gives

$$(b, a) = r_n.$$

Denote the numebr of steps by

$$\#(b, a) = n$$

Question 1: For which number a is

$$\#(34, a)$$

minimal?

For which number a is

$$\#(34, a)$$

maximal? In this case, what is

$$\frac{\#(34, a)}{\ln 34}$$

Question 2: Given $b > a > 0$ with corresponding q_k, $k = 1, \cdots, \#(b, a)$ and $b > \tilde{a} > 0$ with corresponding $\tilde{q}_k$, $k = 1, \cdots, \#(b, \tilde{a})$. Assume

$$\tilde{q}_k > q_k.$$

Show

$$\#(b, a) > \#(b, \tilde{a}).$$

Hint 1: Show that $r_k > \tilde{r}_k$.

Question 3: For which number a is

$$\#(1346269, a)$$

minimal?

For which number a is

$$\#(1346269, a)$$

maximal? In this case, what is

$$\frac{\#(1346269, a)}{\ln 1346269}$$

Question 4: For which number a is

$$\#(63245986, a)$$

minimal?

For which number a is

$$\#(63245986, a)$$

maximal? Explain.

In this case, what is

$$\frac{\#(63245986, a)}{\ln 63245986}$$

Question 5: For which number a is

$$\#(202, a)$$

minimal?

For which number a is

$$\#(202, a)$$

maximal? In this case, what is

$$\frac{\#(202, a)}{\ln 202}$$

Question 6: What is your guess for an upper bound for $\#(b, a)$ in general? Give a motivation.

You have two weeks to finish the report. It should be readable by anybody and just a couple of pages. In essence the report should just answer the six questions.