Remarkable lives an legacy of Sofia Kovalevskaya and Emmy Noether

Leon A. Takhtajan

Stony Brook University, Stony Brook, USA
Euler Mathematical Institute, Saint Petersburg, Russia

ICTS Public Lecture
Bangalore, January 10, 2017

Plan

Plan

Sofia Vasilyevna Kovalevskaya

Sofia Kovalevskaya, 1850–1891

Sofia Kovalevskaya (née **Korvin-Krukovskaya**), was born in Moscow on 15 January 1850. Her father, lieutenant general **Vasily Vasilyevich Korvin-Krukovsky**, was a head of the Moscow artillery and her mother, **Yelizaveta Fedorovna Schubert**, was from a family of German scholars who had settled in Saint Petersburg during the time of Catherine the Great.

Sofia's maternal grandfather was general **Theodor Friedrich von Schubert**, a head of the Russian military topographic service.

Theodor Friedrich von Schubert, 1789–1865

Sofia had a typical upbringing for a girl of her class and time. She was left largely in the care of nurses and governesses, spoke English and French almost as well as she did Russian, and was reared in the belief that her future would be settled by the marriage with a young man of suitable wealth and family position.

Sofia introduction to mathematics

- Sofia's father had a love of science and mathematics and in his student days attended lectures by M.V. Ostrogradsky

Mikhail Vasilyevich Ostrogradsky, 1801–1862

Sofia introduction to mathematics

- Sofia's father had a love of science and mathematics and in his student days attended lectures by M.V. Ostrogradsky

Mikhail Vasilyevich Ostrogradsky, 1801–1862

- The wall of Sofia's room in the family country estate in lieu of the wallpaper were papered with pages by the the lecture notes by Ostrogradsky on differential and integral calculus.

Sofia introduction to mathematics

- Sofia's father had a love of science and mathematics and in his student days attended lectures by M.V. Ostrogradsky

Mikhail Vasilyevich Ostrogradsky, 1801–1862

- The wall of Sofia's room in the family country estate in lieu of the wallpaper were papered with pages by the the lecture notes by Ostrogradsky on differential and integral calculus.

- Gauss-Ostrogradsky theorem

$$\iiint_V (\nabla \cdot \boldsymbol{F}) dV = \oiint_S (\boldsymbol{F} \cdot \boldsymbol{n}) dS$$

Childhood memories

"*As I speak of these, my first contacts with mathematics, I cannot help mentioning a curious circumstance which also helped to arouse my interest in the field. When we moved permanently to the country, the whole house had to be redecorated and all the rooms had to be freshly wallpapered. But since there were many rooms, there wasn't enough wallpaper for one of the nursery rooms... But by happy chance, the paper for this preparatory covering consisted of the lithographed lectures of Professor Ostrogradsky on differential and integral calculus, which my father had acquired as young man. These sheets, all speckled over with strange, unintelligible formulas, soon attracted my attention; I remember as a child standing for hours on end in front of this mysterious wall, trying to figure out at least some isolated sentences and to find the sequence in which the sheets should follow one another. From this protracted daily contemplation, the outer appearance of many of these formulas imprinted themselves in my memory; indeed, their very text left a deep trace in my brain, although they were incomprehensible to me while I was reading them.*"

First mathematics lessons

- "*I took my first lesson in differential calculus from the eminent Petersburg Professor Aleksandr Nikolaevich Strannolyubsky. He was amazed at the speed with which I grasped and assimilated the concepts of limit and of derivatives, exactly as if you knew them in advance. I recall that he expressed himself in just those words. And, as a matter of fact, at the moment when he was explaining these concepts I suddenly had a vivid memory of all this, written on the memorable sheets of Ostrogradsky; and the concept of limit appeared to me as an old friend.*"

First mathematics lessons

- "*I took my first lesson in differential calculus from the eminent Petersburg Professor Aleksandr Nikolaevich Strannolyubsky. He was amazed at the speed with which I grasped and assimilated the concepts of limit and of derivatives, exactly as if you knew them in advance. I recall that he expressed himself in just those words. And, as a matter of fact, at the moment when he was explaining these concepts I suddenly had a vivid memory of all this, written on the memorable sheets of Ostrogradsky; and the concept of limit appeared to me as an old friend.*"

- In 1860th higher equation for women in Russia was not available and one needed to study abroad. Only in 1878 Women's Higher Education Institution (Bestuzhev Courses) were opened in Saint Petersburg with lectures by famous professors Borodin, Mendeleyev, Sechenov and Zelinski.

First mathematics lessons

- "*I took my first lesson in differential calculus from the eminent Petersburg Professor Aleksandr Nikolaevich Strannolyubsky. He was amazed at the speed with which I grasped and assimilated the concepts of limit and of derivatives, exactly as if you knew them in advance. I recall that he expressed himself in just those words. And, as a matter of fact, at the moment when he was explaining these concepts I suddenly had a vivid memory of all this, written on the memorable sheets of Ostrogradsky; and the concept of limit appeared to me as an old friend.*"

- In 1860th higher equation for women in Russia was not available and one needed to study abroad. Only in 1878 Women's Higher Education Institution (Bestuzhev Courses) were opened in Saint Petersburg with lectures by famous professors Borodin, Mendeleyev, Sechenov and Zelinski.

- In order to study abroad, Sofia needed written permission from her father (or husband). Accordingly, she contracted a 'fictitious marriage' with Vladimir Kovalevsky in 1868.

Vladimir Onufrievich Kovalevsky, 1842–1883

- Vladimir Kovalevsky was a Russian revolutioner (belonged to the Russian nihilist movement', a kind of utopian socialism), geologist, paleontologist, founder of evolution paleonthology.

Higher mathematics: Heidelberg and Berlin

- 1869 — Kovalevskaya attended the University of Heidelberg, Germany (was allowed to audit classes with professors approval).

Higher mathematics: Heidelberg and Berlin

- 1869 — Kovalevskaya attended the University of Heidelberg, Germany (was allowed to audit classes with professors approval).
- Same year — went to London with her husband, met Thomas Huxley and Charles Darwin and debated with Herbert Spencer "woman's capacity for abstract thought".
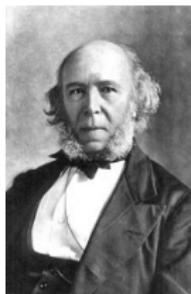
Herbert Spencer, 1820–1903

Higher mathematics: Heidelberg and Berlin

- 1869 — Kovalevskaya attended the University of Heidelberg, Germany (was allowed to audit classes with professors approval).
- Same year — went to London with her husband, met Thomas Huxley and Charles Darwin and debated with Herbert Spencer "woman's capacity for abstract thought".
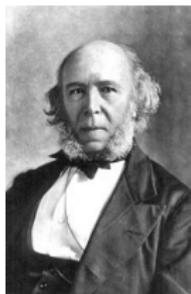
Herbert Spencer, 1820–1903

- 1869-1871 — Kovalevskaya attended lectures by Hermann von Helmholtz and Gustav Kirchhoff.

Higher mathematics: Heidelberg and Berlin

- 1869 — Kovalevskaya attended the University of Heidelberg, Germany (was allowed to audit classes with professors approval).
- Same year — went to London with her husband, met Thomas Huxley and Charles Darwin and debated with Herbert Spencer "woman's capacity for abstract thought".


Herbert Spencer, 1820–1903

- 1869-1871 — Kovalevskaya attended lectures by Hermann von Helmholtz and Gustav Kirchhoff.
- 1871 — took private lessons with Karl Weierstrass in Berlin (the university did not allow to audit classes).

Hermann Ludwig Ferdinand von Helmholtz, 1821–1894
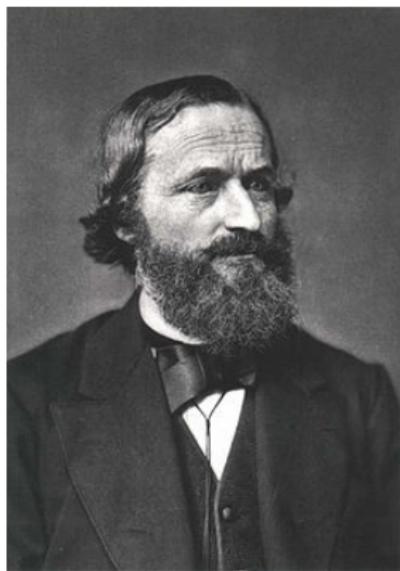
Gustav Robert Kirchhoff, 1824–1887

Weierstraß.

Karl Theodor Wilhelm Weierstrass, 1815–1897

- Called Sofia the most talented of his students (who included such eminent mathematicians as Georg Frobenius, Hermann Schwarz and Carl Runge) and watched out for her interests as carefully as he did for his own.

Doctoral dissertation based on three papers

- Was granted doctoral degree *summa cum laude* from Göttingen University (in absentia) in 1874. Sofia Kovalevskaya was the first woman to be granted a doctoral degree in mathematics.

Doctoral dissertation based on three papers

- Was granted doctoral degree *summa cum laude* from Göttingen University (in absentia) in 1874. Sofia Kovalevskaya was the first woman to be granted a doctoral degree in mathematics.
- Paper on Laplace's calculations on the shape of Saturn rings.

Doctoral dissertation based on three papers

- Was granted doctoral degree *summa cum laude* from Göttingen University (in absentia) in 1874. Sofia Kovalevskaya was the first woman to be granted a doctoral degree in mathematics.
- Paper on Laplace's calculations on the shape of Saturn rings.
- Paper on the reduction of abelian integrals to elliptic integrals.

Doctoral dissertation based on three papers

- Was granted doctoral degree *summa cum laude* from Göttingen University (in absentia) in 1874. Sofia Kovalevskaya was the first woman to be granted a doctoral degree in mathematics.
- Paper on Laplace's calculations on the shape of Saturn rings.
- Paper on the reduction of abelian integrals to elliptic integrals.
- Paper on local existence and uniqueness theorem for Cauchy initial value problem for partial differential equations with analytic coefficients — the celebrated Cauchy-Kovalevskaya theorem.

Doctoral dissertation based on three papers

- Was granted doctoral degree *summa cum laude* from Göttingen University (in absentia) in 1874. Sofia Kovalevskaya was the first woman to be granted a doctoral degree in mathematics.
- Paper on Laplace's calculations on the shape of Saturn rings.
- Paper on the reduction of abelian integrals to elliptic integrals.
- Paper on local existence and uniqueness theorem for Cauchy initial value problem for partial differential equations with analytic coefficients — the celebrated Cauchy-Kovalevskaya theorem.
- Unknown to Weierstrass and Kovalevskaya, Cauchy proved a special case of this theorem in 1842. Starting with Poincaré, Hermite and others it was acknowledged that Kovalevskaya's elegant method proves the general case. She also noted that certain equations have no solutions even when they have "formal power series'" solutions, famous Kovalevskaya example.

Theorem (Cauchy-Kovalevskaya)

Consider the initial value problem

$$\partial_t^m u = G(t, x, \partial_t^j \partial_x^\alpha u), \quad 0 \le j \le m-1; \; j + |\alpha| \le m$$
$$\partial_t^i u(0, x) = g_j(x), \quad 0 \le j \le m-1.$$

Suppose that g_j are real analytic on a neighborhood of $x_0 \in \mathbb{R}^d$, and G is real analytic on a neighborhood of

$$(0, x_0, \partial_t^j \partial_x^\alpha g_j(x_0), \quad 0 \le j \le m-1; \; j + |\alpha| \le m).$$

Then there is a real analytic solution defined on a neighborhood of $(0, x_0) \in \mathbb{R} \times \mathbb{R}^d$. The solution is unique in the sense that if u and v are both real analytic solutions to the equation on a connected neighborhood of $(0, x_0)$, then $u = v$.

Job candidate

- In Russia, the combination of being a woman and of her political views made Sofia unacceptable as a job candidate.

Job candidate

- In Russia, the combination of being a woman and of her political views made Sofia unacceptable as a job candidate.
- In Western Europe, Kovalevskaya's status as Weierstrass's student would have been to her advantage, but her reputation as a nihilist was shocking to European academics, and being a woman was also an obstacle.

Job candidate

- In Russia, the combination of being a woman and of her political views made Sofia unacceptable as a job candidate.
- In Western Europe, Kovalevskaya's status as Weierstrass's student would have been to her advantage, but her reputation as a nihilist was shocking to European academics, and being a woman was also an obstacle.
- The most immediate barrier to her mathematical career, however, was the social convention of the time.

Job candidate

- In Russia, the combination of being a woman and of her political views made Sofia unacceptable as a job candidate.

- In Western Europe, Kovalevskaya's status as Weierstrass's student would have been to her advantage, but her reputation as a nihilist was shocking to European academics, and being a woman was also an obstacle.

- The most immediate barrier to her mathematical career, however, was the social convention of the time.

- Kovalevskaya was married, and married women did not live apart from their husbands, nor did they support themselves with teaching positions.

Job candidate

- In Russia, the combination of being a woman and of her political views made Sofia unacceptable as a job candidate.

- In Western Europe, Kovalevskaya's status as Weierstrass's student would have been to her advantage, but her reputation as a nihilist was shocking to European academics, and being a woman was also an obstacle.

- The most immediate barrier to her mathematical career, however, was the social convention of the time.

- Kovalevskaya was married, and married women did not live apart from their husbands, nor did they support themselves with teaching positions.

- Even Weierstrass thought Kovalevskaya was doing mathematics for the intellectual satisfaction. Her husband would support her and she does not need official recognition of her mathematical accomplishments.

- Only after Kovalevskaya told Weierstrass the true story of her marriage and something of her political beliefs, did he agree that a degree might be useful to her sometime in the future.

- Only after Kovalevskaya told Weierstrass the true story of her marriage and something of her political beliefs, did he agree that a degree might be useful to her sometime in the future.
- Weierstrass persuaded Göttingen to grant her a degree. And only after Vladimir Kovalevsky committed suicide in 1883 did Weierstrass started actively seeking a university position for Kovalevskaya.

- Only after Kovalevskaya told Weierstrass the true story of her marriage and something of her political beliefs, did he agree that a degree might be useful to her sometime in the future.
- Weierstrass persuaded Göttingen to grant her a degree. And only after Vladimir Kovalevsky committed suicide in 1883 did Weierstrass started actively seeking a university position for Kovalevskaya.
- In the meantime there was a hiatus in Kovalevskaya's scientific activity. From 1874, when she returned to Russia, to 1878, while she was pregnant with her only daughter, Kovalevskaya more or less abandoned any attempt at original work in mathematics.

- Only after Kovalevskaya told Weierstrass the true story of her marriage and something of her political beliefs, did he agree that a degree might be useful to her sometime in the future.
- Weierstrass persuaded Göttingen to grant her a degree. And only after Vladimir Kovalevsky committed suicide in 1883 did Weierstrass started actively seeking a university position for Kovalevskaya.
- In the meantime there was a hiatus in Kovalevskaya's scientific activity. From 1874, when she returned to Russia, to 1878, while she was pregnant with her only daughter, Kovalevskaya more or less abandoned any attempt at original work in mathematics.
- In order to teach at the higher levels, one needed a Russian master's degree. But women were forbidden to take the exam to obtain the degree.

- Only after Kovalevskaya told Weierstrass the true story of her marriage and something of her political beliefs, did he agree that a degree might be useful to her sometime in the future.
- Weierstrass persuaded Göttingen to grant her a degree. And only after Vladimir Kovalevsky committed suicide in 1883 did Weierstrass started actively seeking a university position for Kovalevskaya.
- In the meantime there was a hiatus in Kovalevskaya's scientific activity. From 1874, when she returned to Russia, to 1878, while she was pregnant with her only daughter, Kovalevskaya more or less abandoned any attempt at original work in mathematics.
- In order to teach at the higher levels, one needed a Russian master's degree. But women were forbidden to take the exam to obtain the degree.
- She plunged into literary circles, tried her hand at writing, was active in the movement to establish a women's university in St. Petersburg, and so on.

Back to mathematics

Gösta Mittag-Leffler, 1846–1927

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).
- Applications of the theory of elliptic functions (Autumn 1888).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).
- Applications of the theory of elliptic functions (Autumn 1888).
- Theory of partial differential equations (Spring 1890).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).
- Applications of the theory of elliptic functions (Autumn 1888).
- Theory of partial differential equations (Spring 1890).
- Theory of elliptic functions* (Autumn 1889).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).
- Applications of the theory of elliptic functions (Autumn 1888).
- Theory of partial differential equations (Spring 1890).
- Theory of elliptic functions* (Autumn 1889).
- Application of analysis of integer theory (Autumn 1890).

- Kovalevskaya has been offered a position of *privat docent* at Stockholm University in 1883.
- In 1884 she became an editor of *Acta Mathematica*.
- Appointed an *extraordinary professor* for five years in 1884.
- Theory of partial differential equations (Fall 1884).
- Theory of algebraic functions* (Spring 1885).
- Elementary algebra (Spring 1885).
- Theory of Abelian functions* (Fall 1885 to Spring 1887).
- Theory of potential functions (Spring 1886).
- Theory of motion of a rigid body (Fall 1880 and Spring 1887).
- On the curves defined by differential equations, following Poincaré (Autumn 1887 and Spring 1888).
- Theory of theta-functions* (Spring 1888).
- Applications of the theory of elliptic functions (Autumn 1888).
- Theory of partial differential equations (Spring 1890).
- Theory of elliptic functions* (Autumn 1889).
- Application of analysis of integer theory (Autumn 1890).
- * — following Weierstrass.

Permanent secretaries of the Academy to Madam Sofya Kovalevskaya
in Stockholm

Madam,

We have the honour to notify You that the Academy of Sciences
awarded You the Prix Bordin (improvement in an important point of
the theory of motion of a solid body). We invite You, Madam, to be
present at a public session that will be held on Monday, December 24
of this year at one o'clock p.m. exactly and at which time the results
of the competition will be announced in public. We take this
opportunity to convey our personal congratulations and testify to our
confidence of the benefit the Academy anticipates Your work and
Your advances will bring. Madam, receive our assurances in our
highest respect.

L. Pasteur   J. Bertrand

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232
- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232
- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be
- Three integrable cases of the rotation of the heavy rigid body about the fixed point.

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232

- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be

- Three integrable cases of the rotation of the heavy rigid body about the fixed point.

- Euler-Poinsot case (elliptic theta-functions).

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232

- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be

- Three integrable cases of the rotation of the heavy rigid body about the fixed point.

- Euler-Poinsot case (elliptic theta-functions).

- Lagrange-Poisson case (elliptic theta-functions).

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232
- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be
- Three integrable cases of the rotation of the heavy rigid body about the fixed point.
- Euler-Poinsot case (elliptic theta-functions).
- Lagrange-Poisson case (elliptic theta-functions).
- Kovalevskaya case (genus 2 theta-functions).

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232

- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be

- Three integrable cases of the rotation of the heavy rigid body about the fixed point.

- Euler-Poinsot case (elliptic theta-functions).

- Lagrange-Poisson case (elliptic theta-functions).

- Kovalevskaya case (genus 2 theta-functions).

- In 1989 Kovalevskaya was appointed the chair of analysis at Stockholm Univserrsity and was given a lifetime professorship — the first woman in modern times to be so honored.

- "Sur le probleme de la rotation d'un corps solide aulour d'un point fixe"." Acta Math., 1889, t. 12, pp. 177-232
- Submitted anonymously under the maxim:
  Say what you know
  Do what you must
  Whatever will be, will be
- Three integrable cases of the rotation of the heavy rigid body about the fixed point.
- Euler-Poinsot case (elliptic theta-functions).
- Lagrange-Poisson case (elliptic theta-functions).
- Kovalevskaya case (genus 2 theta-functions).
- In 1989 Kovalevskaya was appointed the chair of analysis at Stockholm University and was given a lifetime professorship — the first woman in modern times to be so honored.
- Died in Stockholm in 1891 of pneumonia.

Kovalevskaya case

- Differential equations

$$2\dot{p} = qr, \qquad\qquad \dot{\gamma} = r\gamma' - q\gamma'',$$
$$2\dot{q} = -pr - c\gamma'', \qquad \dot{\gamma}' = p\gamma'' - r\gamma,$$
$$\dot{r} = c\gamma', \qquad\qquad \dot{\gamma}'' = q\gamma - p\gamma'.$$

Kovalevskaya case

- Differential equations

$$2\dot{p} = qr, \qquad\qquad \dot{\gamma} = r\gamma' - q\gamma'',$$
$$2\dot{q} = -pr - c\gamma'', \qquad \dot{\gamma}' = p\gamma'' - r\gamma,$$
$$\dot{r} = c\gamma', \qquad\qquad \dot{\gamma}'' = q\gamma - p\gamma'.$$

- Here the moments of inertia are $I_1 = I_2 = 2I_3$ where $I_3 = 1$, the center of mass is $(x_0, 0, 0)$, $c = Mgx_0$, p, q, r are components of the angular velocitiy and $\gamma, \gamma', \gamma''$ are cosines of angles between the z-axis of fixed coordinate system and axes of coordinate system that is attached to the rigid body and whose origin coincides with the fixed point.

Kovalevskaya case

- Differential equations

$$
\begin{aligned}
2\dot{p} &= qr, & \dot{\gamma} &= r\gamma' - q\gamma'', \\
2\dot{q} &= -pr - c\gamma'', & \dot{\gamma}' &= p\gamma'' - r\gamma, \\
\dot{r} &= c\gamma', & \dot{\gamma}'' &= q\gamma - p\gamma'.
\end{aligned}
$$

- Here the moments of inertia are $I_1 = I_2 = 2I_3$ where $I_3 = 1$, the center of mass is $(x_0, 0, 0)$, $c = Mgx_0$, p, q, r are components of the angular velocitiy and $\gamma, \gamma', \gamma''$ are cosines of angles between the z-axis of fixed coordinate system and axes of coordinate system that is attached to the rigid body and whose origin coincides with the fixed point.

- Kovalevskaya solution uses theta-functions for the genus 2 hyperelliptic curve.

Kovalevskaya case

- Differential equations

$$
\begin{aligned}
2\dot{p} &= qr, & \dot{\gamma} &= r\gamma' - q\gamma'', \\
2\dot{q} &= -pr - c\gamma'', & \dot{\gamma}' &= p\gamma'' - r\gamma, \\
\dot{r} &= c\gamma', & \dot{\gamma}'' &= q\gamma - p\gamma'.
\end{aligned}
$$

- Here the moments of inertia are $I_1 = I_2 = 2I_3$ where $I_3 = 1$, the center of mass is $(x_0, 0, 0)$, $c = Mgx_0$, p, q, r are components of the angular velocitiy and $\gamma, \gamma', \gamma''$ are cosines of angles between the z-axis of fixed coordinate system and axes of coordinate system that is attached to the rigid body and whose origin coincides with the fixed point.

- Kovalevskaya solution uses theta-functions for the genus 2 hyperelliptic curve.

- Solutions are meromorphic functions of complexified time $t \in \mathbb{C}$.

Legacy

- Along with Weierstrass, Hermite, Mittag-Leffler, Picard and Poincaré, Sofia Kovalevskaya was considered one of the best mathematical analysts in Europe.

Pelageya Kochina "*Love and Mathematics: Sofia Kovalevskaya*"
Mir Publishers, Moscow, 1985

Amalie Emmy Noether

Amalie Emmy Noether, 1882–1935

- **Amalie Emmy Noether** was born 23 March 1882 in Erlangen, Bavaria, Germany.

- **Amalie Emmy Noether** was born 23 March 1882 in Erlangen, Bavaria, Germany.
- Her father — **Max Noether** — a well-known mathematician working on algebraic geometry of surfaces (Noether's formula, the Noether inequality).

Max Noether, 1844–1921

- **Amalie Emmy Noether** was born 23 March 1882 in Erlangen, Bavaria, Germany.
- Her father — **Max Noether** — a well-known mathematician working on algebraic geometry of surfaces (Noether's formula, the Noether inequality).

Max Noether, 1844–1921

- Her mother, **Ida Amalia Kaufmann**, the daughter of a wealthy Jewish merchant family.

- In 1900 Noether started her studies at the University of Erlangen.

- In 1900 Noether started her studies at the University of Erlangen.
- One of only two women of 986 students, she was only allowed to audit classes with the permission of individual professors.

- In 1900 Noether started her studies at the University of Erlangen.
- One of only two women of 986 students, she was only allowed to audit classes with the permission of individual professors.
- During the 1903-04 winter semester, Noether studied at the University of Göttingen, attending lectures by the astronomer Karl Schwarzschild and mathematicians Hermann Minkowski, Felix Klein, and David Hilbert.

- In 1900 Noether started her studies at the University of Erlangen.
- One of only two women of 986 students, she was only allowed to audit classes with the permission of individual professors.
- During the 1903-04 winter semester, Noether studied at the University of Göttingen, attending lectures by the astronomer Karl Schwarzschild and mathematicians Hermann Minkowski, Felix Klein, and David Hilbert.
- Restrictions on women's participation were rescinded and Noether returned to the University of Erlangen in the fall 1904 to major in mathematics.

- In 1900 Noether started her studies at the University of Erlangen.
- One of only two women of 986 students, she was only allowed to audit classes with the permission of individual professors.
- During the 1903-04 winter semester, Noether studied at the University of Göttingen, attending lectures by the astronomer Karl Schwarzschild and mathematicians Hermann Minkowski, Felix Klein, and David Hilbert.
- Restrictions on women's participation were rescinded and Noether returned to the University of Erlangen in the fall 1904 to major in mathematics.
- Under the supervision of Paul Gordan in 1907 she wrote her dissertation, "Über die Bildung des Formensystems der ternären biquadratischen Form" ("On complete systems of invariants for ternary biquadratic forms").

- In 1900 Noether started her studies at the University of Erlangen.
- One of only two women of 986 students, she was only allowed to audit classes with the permission of individual professors.
- During the 1903-04 winter semester, Noether studied at the University of Göttingen, attending lectures by the astronomer Karl Schwarzschild and mathematicians Hermann Minkowski, Felix Klein, and David Hilbert.
- Restrictions on women's participation were rescinded and Noether returned to the University of Erlangen in the fall 1904 to major in mathematics.
- Under the supervision of Paul Gordan in 1907 she wrote her dissertation, "Über die Bildung des Formensystems der ternären biquadratischen Form" ("On complete systems of invariants for ternary biquadratic forms").
- In 1908–15 taught at the University of Erlangen's Mathematical Institute without pay.

- In 1915, Noether was invited to the University of Göttingen by David Hilbert and Felix Klein.

- In 1915, Noether was invited to the University of Göttingen by David Hilbert and Felix Klein.
- The philologists and historians objected: "What will our soldiers think when they return to the university and find that they are required to learn at the feet of a woman?"

- In 1915, Noether was invited to the University of Göttingen by David Hilbert and Felix Klein.
- The philologists and historians objected: "What will our soldiers think when they return to the university and find that they are required to learn at the feet of a woman?"
- Hilbert responded with indignation, stating, "I do not see that the sex of the candidate is an argument against her admission as privat dozent. After all, we are a university, not a bath house."

David Hilbert, 1862–1943

Noether theorem

- In 1915 Emmy Noether proved the following theorem (published in 1918):

  Every differentiable symmetry of the action of a physical system has a corresponding conservation law.

Noether theorem

- In 1915 Emmy Noether proved the following theorem (published in 1918):

  Every differentiable symmetry of the action of a physical system has a corresponding conservation law.

- Applications include conservation of energy, momentum and angular momentum in classical mechanics, energy-momentum tensor in electromagnetism, special and general relativity, charge conservation, etc.

Noether theorem

- In 1915 Emmy Noether proved the following theorem (published in 1918):

  Every differentiable symmetry of the action of a physical system has a corresponding conservation law.

- Applications include conservation of energy, momentum and angular momentum in classical mechanics, energy-momentum tensor in electromagnetism, special and general relativity, charge conservation, etc.

- "Noether theorem certainly is one of the most important mathematical theorems ever proved in guiding the development of modern physics, possibly on a par with the Pythagorean theorem" (in "Symmetry and the Beautiful Universe" by L.M. Lederman and C.T. Hill).

Noether theorem

- In 1915 Emmy Noether proved the following theorem (published in 1918):

  Every differentiable symmetry of the action of a physical system has a corresponding conservation law.

- Applications include conservation of energy, momentum and angular momentum in classical mechanics, energy-momentum tensor in electromagnetism, special and general relativity, charge conservation, etc.

- "Noether theorem certainly is one of the most important mathematical theorems ever proved in guiding the development of modern physics, possibly on a par with the Pythagorean theorem" (in "Symmetry and the Beautiful Universe" by L.M. Lederman and C.T. Hill).

- In classical physics Noether theorem uses equations of motion while its analog in quantum physics deals with expectation values of quantum fields.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

there is n such that $I_n = I_{n+1} = \cdots$.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

  there is n such that $I_n = I_{n+1} = \cdots$.

- **Noetherian module** is a module that satisfies the ascending chain condition on its submodules.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

there is n such that $I_n = I_{n+1} = \cdots$.

- **Noetherian module** is a module that satisfies the ascending chain condition on its submodules.

- **Noetherian scheme** is a scheme that admits a finite covering by open affine subsets $\mathrm{Spec}(A_i)$, where A_i are Noetherian rings.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

  there is n such that $I_n = I_{n+1} = \cdots$.

- **Noetherian module** is a module that satisfies the ascending chain condition on its submodules.

- **Noetherian scheme** is a scheme that admits a finite covering by open affine subsets $\mathrm{Spec}(A_i)$, where A_i are Noetherian rings.

- **Noether normalization lemma** — an important step in proving Hilbert's Nullstellensatz.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

there is n such that $I_n = I_{n+1} = \cdots$.

- **Noetherian module** is a module that satisfies the ascending chain condition on its submodules.

- **Noetherian scheme** is a scheme that admits a finite covering by open affine subsets $\text{Spec}(A_i)$, where A_i are Noetherian rings.

- **Noether normalization lemma** — an important step in proving Hilbert's Nullstellensatz.

- The **Lasker-Noether theorem**: in a Noetherian ring every ideal can be decomposed as a product of finitely many primary ideals.

Abstract algebra

- **Noetherian ring** — a ring that satisfies the ascending chain condition on ideals: for any such chain of ideals

$$I_1 \subseteq I_2 \cdots \subseteq I_{k-1} \subseteq I_k \subseteq I_{k+1} \subseteq \cdots$$

there is n such that $I_n = I_{n+1} = \cdots$.

- **Noetherian module** is a module that satisfies the ascending chain condition on its submodules.

- **Noetherian scheme** is a scheme that admits a finite covering by open affine subsets $\mathrm{Spec}(A_i)$, where A_i are Noetherian rings.

- **Noether normalization lemma** — an important step in proving Hilbert's Nullstellensatz.

- The **Lasker-Noether theorem**: in a Noetherian ring every ideal can be decomposed as a product of finitely many primary ideals.

- Emmy Noether "changed the face of algebra by her work" (Hermann Weyl).

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.
- She has 16 doctoral students.

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.
- She has 16 doctoral students.
- In the winter of 1928-29 Noether was lecturing at the Moscow State University and collaborating with P.S. Aleksandrov, N.G. Chebotatev and L.S. Pontryagin. She planned to return to Moscow.

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.
- She has 16 doctoral students.
- In the winter of 1928-29 Noether was lecturing at the Moscow State University and collaborating with P.S. Aleksandrov, N.G. Chebotatev and L.S. Pontryagin. She planned to return to Moscow.
- In 1932 Emmy Noether and Emil Artin received the Ackermann-Teubner Memorial Award for their contributions to mathematics. Nevertheless, she was not elected to the Göttingen Academy of Sciences and was never promoted to the position of a full professor.

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.
- She has 16 doctoral students.
- In the winter of 1928-29 Noether was lecturing at the Moscow State University and collaborating with P.S. Aleksandrov, N.G. Chebotatev and L.S. Pontryagin. She planned to return to Moscow.
- In 1932 Emmy Noether and Emil Artin received the Ackermann-Teubner Memorial Award for their contributions to mathematics. Nevertheless, she was not elected to the Göttingen Academy of Sciences and was never promoted to the position of a full professor.
- In 1933 when Nazi came to power in Germany, Noether was expelled from the University and moved to the US to teach in the Bryn Mawr College, PA (a private women's liberal arts college).

- After the fall of German Empire in November 1918, Noether was appointed a privat dozent in the University of Göttingen in 1919 and untenured professor in 1922.
- She has 16 doctoral students.
- In the winter of 1928-29 Noether was lecturing at the Moscow State University and collaborating with P.S. Aleksandrov, N.G. Chebotatev and L.S. Pontryagin. She planned to return to Moscow.
- In 1932 Emmy Noether and Emil Artin received the Ackermann-Teubner Memorial Award for their contributions to mathematics. Nevertheless, she was not elected to the Göttingen Academy of Sciences and was never promoted to the position of a full professor.
- In 1933 when Nazi came to power in Germany, Noether was expelled from the University and moved to the US to teach in the Bryn Mawr College, PA (a private women's liberal arts college).
- In 1935 Emmy Noether died and was buried in the walkway surrounding the cloisters of Bryn Mawr's Library.

Tribute to Emmy Noether

$$S = \int \mathscr{L}(\varphi, \partial_\mu \varphi) d^4 x.$$

Principle of the least action: $\delta S = 0$ yields Euler-Lagrange equations

$$\frac{\partial \mathscr{L}}{\partial \varphi} - \frac{\partial}{\partial x^\mu} \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi)} = 0.$$

Energy-momentum tensor:

$$\theta^\mu_\nu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi)} \partial_\nu \varphi - \delta^\mu_\nu \mathscr{L}.$$

<p style="text-align:center; color:red;">Noether theorem</p>

If the action S is invariant under the infinitesimal transformations $\Delta x^\mu = X^\mu_\nu \varepsilon^\nu$ and $\Delta \varphi = \Phi_\mu \varepsilon^\mu$, then the Noether current

$$J^\mu_\nu = \frac{\partial \mathscr{L}}{\partial(\partial_\mu \varphi)} \Phi_\nu - \theta^\mu_\lambda X^\lambda_\nu$$

is conserved:

$$\partial_\mu J^\mu_\nu = 0.$$