IL NUOVO CIMENTO VOL. XXXVII, N. 2 16 Maggio 1965

A No-Interaction Theorem in Classical Relativistic Hamiltonian Particle Mechanics.

H. LEUTWYLER

Institut für Theoretische Physik der Universität - Bern

(ricevuto il 26 Novembre 1964)

Summary. — It is shown that a nondegenerate Hamiltonian theory of a finite number of classical particles cannot describe any interaction if the theory is relativistically invariant and if the co-ordinates of the particles transform correctly under the inhomogeneous Lorentz group.

1. – Introduction.

CURRIE, JORDAN and SUDARSHAN (1) have shown that a Hamiltonian theory of two classical particles is unable to describe any interaction if the principle of relativity is satisfied, *i.e.* if

a) there exists a set of ten generators of the inhomogeneous Lorentz group, and

b) the observables of the theory (the co-ordinates of the particles) transform correctly under the inhomogeneous Lorentz group.

CANNON and JORDAN (2) have established the analogous no-interaction theorem for three particles.

On the other hand, if one allows an infinite number of degrees of freedom, examples of Hamiltonian theories which satisfy the principle of relativity and which describe interaction are given by local field theories.

(1) D. G. CURRIE, T. F. JORDAN and E. C. G. SUDARSHAN: *Rev. Mod. Phys.*, **35**, 350 (1963). Of course the method of introducing relativistic symmetry by constructing a set of generators has been used earlier; see the above review article for references.

(2) J. T. CANNON and T. F. JORDAN: to be published.

The question remained: Is it necessary to have an infinite number of degrees of freedom in order that a Hamiltonian system which satisfies the principle of relativity is able to describe interaction or is some finite number of degrees of freedom > 3 sufficient?

The purpose of the present paper is to show that the no-interaction theorem is in fact valid for any finite number of particles.

We shall denote the generators of time translations, space translations space rotations and pure Lorentz transformations by H, P_i, J_i, K_i, respectively ($i = 1, 2, 3$). These generators are functions of the $6n$ canonical variables q_i^a, p_i^a ($a = 1, \dots, n$).

The Poisson bracket relations characteristic of the Poincaré group read [1]

$$\text{(1.1)} \qquad \begin{cases} [H, P_i] = 0\,, & [H, J_i] = 0\,, & [H, K_i] = -P_i\,, \\ [P_i, P_k] = 0\,, & [P_i, J_k] = \varepsilon_{ikl} P_l\,, & [P_i, K_k] = -\delta_{ik} H\,, \\ [J_i, J_k] = \varepsilon_{ikl} J_l\,, & [J_i, K_k] = \varepsilon_{ikl} K_l\,, & [K_i, K_k] = -\varepsilon_{ikl} J_l\,. \end{cases}$$

The requirement that the particle co-ordinates transform correctly under Lorentz transformations is equivalent to [1]

$$\text{(1.2)} \qquad \begin{cases} [q_i^a, P_k] = \delta_{ik}\,, \\ [q_i^a, J_k] = -\varepsilon_{ikl} q_l^a\,, \\ [q_i^a, K_k] = q_i^a [q_k^a, H]\,. \end{cases} \qquad (a = 1, \dots, n)$$

We make no assumptions about the transformation properties of the momenta.

THEOREM: If the set of ten functions H, P_i, J_i and K_i satisfies the bracket relations (1.1) and (1.2) and if the equations of motion are not degenerate, *i.e.*

$$\text{(1.3)} \qquad \det \frac{\partial^2 H}{\partial p_i^a \, \partial p_k^b} \neq 0\,,$$

then the acceleration of each particle vanishes

$$\text{(1.4)} \qquad \big[[q_i^a, H], H\big] = 0 \qquad (a = 1, \dots, n).$$

The proof is carried out in two steps. First, we show that the generators P_i and J_i may be brought to their free-particle form

$$\text{(1.5)} \qquad P_i^0 = \sum_a p_i^a\,, \qquad J_i^0 = \sum_a \varepsilon_{ikl} q_k^a p_l^a\,,$$

3273

if one performs a suitable canonical transformation which does not affect the co-ordinates q_i^a, but only the momenta. The co-ordinates are not allowed to be transformed, because they have a physical meaning as the positions of the particles.

This part of the problem has already been solved by CANNON and JORDAN (2). We present here a simplified version of their demonstration which is based on a generalization of the theorem of LOMONT and MOSES (3) which is proved by group-theoretical methods in the preceding paper.

The second step is to analyse the remaining four generators H and K_i. It is shown that they may be brought to their free-particle form

$$(1.6)\qquad \begin{cases} H^0 = \sum_a (p_a^2 + m_a^2)^{\frac{1}{2}}\,, \\ K_i^0 = \sum_a (p_a^2 + m_a^2)^{\frac{1}{2}} q_a^i\,, \end{cases}$$

by means of a similar canonical transformation which—besides leaving the co-ordinates unchanged—does not modify P_i and J_i.

Of course (1.6) implies (1.4).

2. – The subgroup of space translations and rotations.

In this Section we analyse the subgroup of space translations and rotations. We want to show that the generators P_i and J_i of this group may be brought to their free-particle forms by means of a canonical transformation.

It is easy to see from the bracket relations of the generators with the co-ordinates, eq. (1.2), that J_i and P_k differ from their free-particle forms (1.4) by functions of the q variables only

$$(2.1)\qquad \begin{cases} J_i = J_i^0 + F_i(q)\,, \\ P_i = P_i^0 + W_i(q)\,. \end{cases}$$

On the other hand the restrictions imposed on the functions F_i and W_i by the bracket relations between the generators themselves are not trivial. These restrictions coincide precisely with eq. (5.2) of the preceding paper, where it is shown that they imply

$$(2.2)\qquad \begin{cases} F_i = [J_i, F]\,, \\ W_i = [P_i, F]\,, \end{cases}$$

where F is some function of the q variables only.

---

(3) J. S. LOMONT and H. E. MOSES: *Comm. Pure Appl. Math.*, **14**, 69 (1961); J. B. KELLER: *Comm. Pure Appl. Math.*, **14**, 77 (1961).

This result invites us to perform a canonical transformation

$$\begin{cases} q_a^{i'} = q_a^i \,, \\ p_a^{i'} = p_a^i - \dfrac{\partial F}{\partial q_a^i} \,, \end{cases} \tag{2.3}$$

which in fact reduces both J_i and P_i to their free-particle form. From now on we shall work with the new canonical variables only and drop the primes.

3. – The brackets linear in K_i and H.

In this Section we briefly review some results obtained by CANNON and JORDAN (2).

The transformation properties of the co-ordinates under pure Lorentz transformations, expressed by

$$[q_i^a, K_k] = q_k^a [q_i^a, H] \,, \tag{3.1}$$

imply

$$(q_k^a - q_k^b) \frac{\partial^2 H}{\partial p_i^a \partial p_i^b} = 0 \tag{3.2}$$

and therefore

$$H = \sum_a h^a(p^a, q) \,, \tag{3.3}$$

where each term h^a is independent of the momenta except of p_i^a. The bracket relation

$$[P_i, H] = 0 \tag{3.4}$$

states that H is translation-invariant.

It is easy to see that the decomposition (3.3) may be arranged so that each term h^a is translation-invariant. For this purpose we may take

$$\overline{h^a}(p^a, q) = h^a(p^a, q - q^1) \,. \tag{3.5}$$

Clearly $\bar{h}^a$ is translation invariant and $\sum \bar{h}^a$ is identical with $\sum h^a$.

The relation

$$[J_i, H] = 0 \tag{3.6}$$

is the infinitesimal form of

$$\sum_a h^a(Rp^a, Rq) = \sum_a h^a(p, q) \,, \tag{3.7}$$

where R is a rotation. Let us average (3.7) over the rotation group and put

$$\bar{h}^a(p^a, q) = \int h^a(Rp^a, Rq)\, d\mu(R) . \tag{3.8}$$

Due to the invariance of the measure $d\mu(R)$, $\bar{h}^a$ is rotation-invariant. Since $\sum h^a = \sum \bar{h}^a$ we may replace h^a by $\bar{h}^a$ such that each of the terms becomes rotation-invariant. Clearly the translational invariance of the individual terms is not affected by this replacement.

Finally inserting the form (3.3) for H in (3.1) one finds

$$K_i = \sum_a h^a q_i^a + k_i , \tag{3.9}$$

where k_i is a translation-invariant vector function independent of the momenta.

4. – Brackets quadratic in K_j and H.

Evaluating the bracket relations

$$[K_j, H] = P_j , \tag{4.1}$$

one obtains

$$\frac{1}{2}\sum_a \frac{\partial}{\partial p_i^a}(h^{a^2} - p^{a^2}) + \sum_{a,k} \frac{\partial k_i}{\partial q_k^a}\frac{\partial h^a}{\partial p_k^a} + \sum_{a,b,k} \frac{\partial h^b}{\partial q_k^a}\frac{\partial h^a}{\partial p_k^a}(q_i^b - q_i^a) = 0 . \tag{4.2}$$

In this Section we prove that (4.2) implies

$$\frac{\partial h^a}{\partial q_i^b} = \sum_k \frac{\partial^2 L}{\partial q_i^b \partial q_k^a}\frac{\partial h^a}{\partial p_k^a} + \frac{\partial M^a}{\partial q_i^b} , \tag{4.3}$$

where L and M^a are functions of the co-ordinates only, invariant with respect to translations and rotations.

To obtain this result let us take the second derivative of (4.2) with respect to p_l^c, p_m^d, where $c \neq d$. The first two terms give no contribution and from the last term we obtain

$$(q_i^c - q_i^d)\left\{\sum_k \frac{\partial^2 h^c}{\partial q_k^d \partial p_l^c}\frac{\partial^2 h^d}{\partial p_k^d \partial p_m^d} - \sum_k \frac{\partial^2 h^d}{\partial q_k^c \partial p_m^d}\frac{\partial^2 h^c}{\partial p_k^c \partial p_l^c}\right\} = 0 .$$

Since $c \neq d$ the curly bracket must vanish. In order to solve this differential equation for the functions h^a we invoke the nondegeneracy of the equations of motion assumed in the statement of the no-interaction theorem. The requirement (1.3) guarantees that the equations

$$\dot{q}_i^a = \frac{\partial H}{\partial p_i^a} = \frac{\partial h^a}{\partial p_i^a} \tag{4.4}$$

may be solved for p_i^a. Therefore the individual determinants

$$\det \frac{\partial^2 h^a}{\partial p_i^a \partial p_k^a} \qquad (a = 1, \ldots, n) \tag{4.5}$$

must be different from zero.

In other words the 3×3 matrix $\partial^2 h^a / \partial p_i^a \partial p_k^a$ has an inverse which we denote by h_{ik}^{a-1}. With the help of this inverse (4.3) may be separated

$$\sum_l \frac{\partial^2 h^c}{\partial q_k^d \partial p_l^c} h_{li}^{c-1} = \sum_l \frac{\partial^2 h^d}{\partial q_i^c \partial p_l^d} h_{lk}^{d-1} \qquad (c \neq d)\,. \tag{4.6}$$

Since the left-hand side is independent of p_m^d the right-hand side must be independent as well, *i.e.* it is a function of the co-ordinates only, say λ_{ik}^{cd} which is symmetric under the simultaneous interchange of c, d and i, k:

$$\lambda_{ik}^{cd} = \lambda_{ki}^{dc}\,, \tag{4.7}$$

$$\frac{\partial^2 h^c}{\partial q_i^d \partial p_k^c} = \sum_l \lambda_{il}^{dc} \frac{\partial^2 h^c}{\partial p_l^c \partial p_k^c}\,. \tag{4.8}$$

This equation may be integrated immediately to give

$$\frac{\partial h^c}{\partial q_i^d} = \sum_l \lambda_{il}^{dc} \frac{\partial h^c}{\partial p_l^c} + \mu_i^{dc}\,, \tag{4.9}$$

where μ_i^{dc} is a function of the co-ordinates only. Note that we assumed $c \neq d$ to derive (4.7)–(4.9). However, as shown in the Appendix, the validity of these equations may be extended to include $c = d$ by making use of the invariance of h^a under rotations and translations.

The integrability condition for the differential eq. (4.9) reads

$$\sum_l \left\{ \frac{\partial \lambda_{il}^{dc}}{\partial q_k^a} - \frac{\partial \lambda_{kl}^{ac}}{\partial q_i^d} \right\} \frac{\partial h^c}{\partial p_l^c} + \frac{\partial \mu_i^{dc}}{\partial q_k^a} - \frac{\partial \mu_k^{ac}}{\partial q_i^d} = 0\,. \tag{4.10}$$

Taking the derivative with respect to p_m^c and making use of the inverse h_{ml}^{c-1}

one finds

$$(4.11)\qquad \begin{cases} \dfrac{\partial \lambda^{dc}_{lk}}{\partial q^a_i} - \dfrac{\partial \lambda^{ac}_{ik}}{\partial q^d_l} = 0\,, \\[2ex] \dfrac{\partial \mu^{dc}_l}{\partial q^a_i} - \dfrac{\partial \mu^{ac}_i}{\partial q^d_l} = 0\,. \end{cases}$$

Therefore

$$(4.12)\qquad \begin{cases} \lambda^{dc}_{ki} = \dfrac{\partial L^c_i}{\partial q^d_k}\,, \\[2ex] \mu^{dc}_i = \dfrac{\partial M^c}{\partial q^d_i}\,. \end{cases}$$

Furthermore the symmetry (4.7) leads to

$$(4.13)\qquad L^c_i = \frac{\partial L}{\partial q^c_i}\,.$$

This establishes the result (4.3). What remains to be shown is that L and M^a may be chosen to be invariant with respect to translations and rotations.

Let us investigate the invariance with respect to translations first. As may be seen from (4.8) and (4.9) the invariance of h^c requires

$$(4.14)\qquad \sum_d \lambda^{dc}_{il} = 0\,, \qquad \sum_d \mu^{dc}_i = 0\,.$$

By virtue of (4.12) this states that L^c_i and M^c are invariant. Clearly L is not determined uniquely by λ^{dc}_{il}, but only up to a linear function of the co-ordinates. The problem is to show that this arbitrariness may be adjusted in such a way that L becomes translation-invariant. Let us put

$$(4.15)\qquad \tilde{L} = L + \sum_{a,k} C^a_k q^a_k\,,$$

$$(4.16)\qquad \tilde{L}^a_k = L^a_k + C^a_k\,.$$

It is shown in the Appendix that

$$(4.17)\qquad L_k = \sum_c L^c_k$$

is a constant. Therefore, if we choose

$$(4.18)\qquad C^a_k = -\frac{1}{n}\, L_k\,,$$

then

$$\sum_a L^a_k = \sum_a \frac{\partial}{\partial q^a_k} L = 0 , \tag{4.19}$$

i.e L is translation-invariant.

The invariance of h^c under rotations determines the transformation properties of λ^{dc}_{il} and μ^{dc}_i through (4.8) and (4.9)

$$\lambda^{dc}_{il}(R^{-1}q) = R^k_i R^m_l \lambda^{dc}_{km}(q) , \tag{4.20}$$

$$\mu^{dc}_i(R^{-1}q) = R^k_i \mu^{dc}_k(q) . \tag{4.21}$$

Equation (4.21) is equivalent to the statement that

$$\Delta^c(R) = M^c(Rq) - M^c(q) \tag{4.22}$$

is independent of q. It is easy to see that by the definition of $\Delta^c(R)$ the quantity $\exp[\Delta^c(R)]$ furnishes a one-dimensional representation of the rotation group. Since there are no nontrivial one-dimensional representations of the rotation group we have $\Delta^c(R)=0$ and therefore $M^c(q)$ is rotation-invariant. To show that $L(q)$ may be chosen to be rotation-invariant put

$$\bar{L}(q) = \int \tilde{L}(Rq)\, \mathrm{d}\mu(R) , \tag{4.23}$$

where $\mathrm{d}\mu(R)$ is the invariant measure and the integration extends over the full rotation group; $\tilde{L}$ is the translation-invariant function constructed previously. Clearly $\bar{L}$ is translation-invariant. Furthermore

$$\frac{\partial^2 \bar{L}(q)}{\partial q^a_i \partial q^b_k} = \int \frac{\partial^2 L(Rq)}{\partial (Rq)^a_l \partial (Rq)^b_m} R^i_l R^k_m \,\mathrm{d}\mu(R) = \int \lambda^{ab}_{lm}(Rq) R^i_l R^k_m \,\mathrm{d}\mu(R) = \lambda^{ab}_{ik}(q) \tag{4.24}$$

due to the transformation property (4.20) of λ^{ab}_{ik}. This proves that L may be replaced by $\bar{L}$ which is invariant with respect to translations as well as rotations.

5. – Free-particle form for K_i and H.

What remains to be done is to solve the differential eq. (4.3). The simplest way to do this is to perform a canonical transformation

$$\begin{cases} q^{a'}_i = q^a_i , \\ p^{a'}_i = p^a_i - \dfrac{\partial L}{\partial q^a_i} . \end{cases} \tag{5.1}$$

With

$$h^{a'}(q', p') = h^a(q, p) , \qquad M^{a'}(q') = M^a(q) , \tag{5.2}$$

eq. (4.3) reduces to

$$\frac{\partial h^{a'}}{\partial q_i^{b'}} = \frac{\partial M^{a'}}{\partial q_i^{b'}} . \tag{5.3}$$

Therefore

$$h^{a'}(q', p') = M^{a'}(q') + N^{a'}(p^{a'}) , \tag{5.4}$$

i.e. in terms of the new variables h^a separates into a function M^a of the coordinates only and a function N^a of the momentum p_i^a only. The canonical transformation (5.1) leaves the free-particle forms of both J_i and P_i invariant since L is invariant with respect to both rotations and translations.

From now on we shall work with the new variables only and drop the primes. Let us return to eq. (4.2) which was the basis of the analysis is Sect. **4**. In terms of the functions N^a and M^a defined by (5.4) this equation reads

$$\frac{1}{2} \sum_a \frac{\partial}{\partial p_i^a} (N^{a^2} - p^{a^2}) + \sum_a \frac{\partial N^a}{\partial p_k^a} C_{ik}^a = 0 , \tag{5.5}$$

where

$$C_{ik}^a = \frac{\partial}{\partial q_k^a} \left(\sum_b M^b q_i^b + k_i\right) - q_i^a \frac{\partial}{\partial q_k^a} \sum_b M^b . \tag{5.6}$$

It is easy to see that C_{ik}^a is a constant by taking the second derivative of (5.5) with respect to p_k^a, q_l^b. Furthermore, since M^b is rotation-invariant and k_i transforms like a vector under rotations, C_{ik}^a has the transformation properties of a second-rank tensor. As is well known the only numerically invariant second-rank tensor is given by δ_{ik}, therefore

$$C_{ik}^a = C^a \delta_{ik} \tag{5.7}$$

and (5.5) becomes

$$\sum_a \frac{\partial}{\partial p_i^a} \{(N^a + C^a)^2 - p^{a^2}\} = 0 . \tag{5.8}$$

Therefore

$$\frac{\partial^2}{\partial p_i^a \partial p_k^a} \{(N^a + C^a)^2 - p^{a^2}\} = 0 , \tag{5.9}$$

i.e.

$$(N^a + C^a)^2 - p^{a^2} = d_k^a p_k^a + d^a . \tag{5.10}$$

The rotational invariance of h^a and M^a implies that N^a is invariant; therefore d_k^a must be a numerically invariant sector. Since no such object exists,

d_k^a vanishes. Hence

(5.11) $$N^a + C^a = (p^{a^2} + d^a)^{\frac{1}{2}} .$$

Finally, to analyse the form of $\sum_a M^a$, let us introduce the notation

(5.12) $$\begin{cases} f = \sum_a M^a , \\ f_i = \sum_a M^a q_i^a + k_i . \end{cases}$$

In terms of these quantities eqs. (5.6) take the form

(5.13) $$C^a \delta_{ik} = \frac{\partial f_i}{\partial q_k^a} - q_i^a \frac{\partial f}{\partial q_k^a} .$$

Consider this as a total system of differential equations for the functions f_i. The integrability conditions for this system read

(5.14) $$\delta_{ab}\left(\delta_{li}\frac{\partial f}{\partial q_k^a} - \delta_{ki}\frac{\partial f}{\partial q_l^b}\right) + (q_i^a - q_i^b)\frac{\partial^2 f}{\partial q_k^a \partial q_l^b} = 0 .$$

If $a = b$ and $l = i \neq k$ one finds

(5.15) $$\frac{\partial f}{\partial q_k^a} = 0 .$$

Therefore f is a constant and

(5.16) $$f_i = \sum_a C^a q_i^a + e_i .$$

In order that f_i transforms like a vector under rotations, e_i must vanish. Therefore

(5.17) $$k_i = \sum_a (C^a - M^a) q_i^a .$$

Since k_i and M^a are translation-invariant we must have

(5.18) $$\sum_a (C^a - M^a) = 0 .$$

Therefore

(5.19) $$H = \sum_a (N^a + M^a) = \sum_a (p_a^2 + d_a)^{\frac{1}{2}}$$

and

(5.20) $$K_i = \sum_a h^a q_i^a + k_i = \sum_a (p_a^2 + d_a)^{\frac{1}{2}} q_i^a .$$

The requirement that the particle velocities

$$\dot{q}_i^a = \frac{\partial H}{\partial p_i^a} = p_i^a(p_a^2 + d_a)^{-\frac{1}{2}} \tag{5.21}$$

be smaller than the velocity of light implies

$$d_a > 0 .$$

Therefore we may write

$$d_a = m_a^2 \tag{5.22}$$

and all ten generators H, P_i, K_i, J_i are brought to their usual free-particle form.

* * *

This investigation originated in stimulating discussions with Prof. E. C. G. SUDARSHAN and Dr. T. F. JORDAN and the author wishes to express his thanks for their interest in this work.

APPENDIX

The derivation of the differential eq. (4.9) and the symmetry relation (4.7) was based on the assumption $c \neq d$. An immediate generalization of (4.8) to the case $c = d$ is obtained by observing that, due to translational invariance of h^c

$$\sum_d \frac{\partial^2 h^c}{\partial q_i^d \partial p_k^c} = 0 . \tag{A.1}$$

Therefore

$$\frac{\partial^2 h^c}{\partial q_i^c \partial p_k^c} = -\sum_{d \neq c} \frac{\partial^2 h^c}{\partial q_i^d \partial p_k^c} = \sum_l \lambda_{il}^{cc} \frac{\partial^2 h^c}{\partial p_l^c \partial p_k^c} , \tag{A.2}$$

where

$$\lambda_{il}^{cc} = -\sum_{d \neq c} \lambda_{il}^{dc} . \tag{A.3}$$

Consequently (4.9)–(4.12) are true for $c = d$ as well.

Somewhat more involved is the extension of the symmetry relation (4.7) to the case $c = d$. Let us write

$$\frac{\partial L_k^c}{\partial q_l^d} - \frac{\partial L_l^d}{\partial q_k^c} = \delta^{cd} \Lambda_{kl}^c \tag{A.4}$$

3282

instead of (4.7) to include $c = d$. According to the definition of λ_{ik}^{cc} we have

$$\sum_c \lambda_{ik}^{cd} = 0\,, \tag{A.5}$$

which is equivalent to the statement that L_k^d is translation-invariant. If (A.4) is summed with respect to c one therefore finds

$$\Lambda_{ik}^d = \frac{\partial L_k}{\partial q_i^d}\,, \qquad L_k = \sum_c L_k^c\,. \tag{A.6}$$

On the other hand the integrability conditions for (A.4) read

$$\frac{\partial}{\partial q_i^a}(\delta^{cd}\Lambda_{kl}^c) + \frac{\partial}{\partial q_k^c}(\delta^{da}\Lambda_{li}^d) + \frac{\partial}{\partial q_l^d}(\delta^{ac}\Lambda_{ik}^a) = 0\,. \tag{A.7}$$

If $c = d$ this implies, by virtue of the antisymmetry of Λ_{ik}^c,

$$\frac{\partial \Lambda_{kl}^c}{\partial q_i^a} = 0\,. \tag{A.8}$$

Therefore Λ_{kl}^c is a constant.

Finally let us make use of the invariance of h^a under rotations. According to (4.8) λ_{il}^{dc} has the transformation properties of a tensor, *i.e.*

$$\lambda_{il}^{dc}(R^{-1}q) = R_i^k R_l^m \lambda_{km}^{dc}(q)\,. \tag{A.9}$$

Therefore Λ_{kl}^c must be a tensor with the same transformation properties. Since it is independent of the co-ordinates, it represents a numerically invariant antisymmetric tensor of second rank. As is well known there is no such tensor for the rotation group

$$\Lambda_{lm}^d = 0\,, \tag{A.10}$$

the symmetry relation (4.7) is therefore valid even if $c = d$. Equation (A.10) implies furthermore that L_k defined in (A.6) is a constant; this result is needed in Sect. **4** in order to demonstrate that L may be chosen to be translation-invariant.

RIASSUNTO (*)

Si dimostra che una teoria hamiltoniana non degenere di un numero finito di particelle classiche non può descrivere nessuna interazione se la teoria è relativisticamente invariante e se le coordinate delle particelle si trasformano correttamente rispetto al gruppo di Lorentz omogeneo.

(*) *Traduzione a cura della Redazione.*