MAT 536 SPRING 2021 HOMEWORK 13

More challenging problems are marked by *.

1. Suppose $f(z)$ is a complex-valued harmonic function on domain D. Prove that if $|f(z)|$ is constant in D, then $f(z)$ is constant in D.

2. Suppose that both $f(z)$ and $zf(z)$ are complex-valued harmonic functions on D. Prove that $f(z)$ is holomorphic.

3. Prove that if a sequence of positive harmonic functions on D converges pointwise, then it converges uniformly on compact subsets of D.

4. Problem **4** on p. 171 in Ahlfors.

5. Problem **5** on p. 171 in Ahlfors.

6. Problem **6** on p. 171 in Ahlfors.

7*. Problem **5** on p. 174 in Ahlfors