# MAT 535 SPRING 2023 REVIEW FOR THE FINAL EXAM

## General

The exam will be in class on Tuesday, May 9, 8:00am - 10:45am. It will consist of 6-7 problems and will be a closed book exam.

The problems will test the basic understanding of the course material. Solutions should be concise, with clear indication which result from the textbook or proved in class you are using. The exam will cover the following material, based on the lectures and Ch. 11, Ch. 13 (§13.3 is omitted), Ch. 14 (§14.7 and §14.9 are omitted), Ch. 18, §18.1, §18.3. The final exam will not include topics from homological algebra and category theory.

## Linear and multilinear algebra

1) Hermitian and Euclidean vector spaces.
2) Schur's theorem for operators in complex vector spaces.
3) Spectral theorem for unitary, self-adjoint and normal operators and analogous operators in Euclidean vector spaces.
4) Bilinear and quadratic forms, symmetric, alternating and Hermitian forms.
5) Tensor, symmetric and exterior algebras of a module over a commutative ring with 1. Operators $A^{\otimes k}$, $\mathrm{Sym}^k(A)$ and $\wedge^k A$ for$A \in \mathrm{End}V$, where $V$ is a vector space.

## Field theory

1) Field extensions, adjoining roots.
2) Splitting fields, algebraic closure.
3) Separable and inseparable extensions.
4) Finite fields.
5) Cyclotomic extensions and polynomials.
6) The primitive element theorem.

## Galois theory

1) Galois extensions; a definition as in Dummit & Foote and as in Lang.
2) Three basic facts:

(a) The group $\mathrm{Aut}(K/F)$ permutes the roots of irreducible $f(x) \in F[x]$.
(b) $|\mathrm{Aut}(K/F)| \leq [E : F]$, where $E/F$ is a splitting field of $f(x) \in F[x]$, with the equality if $f(x)$ is separable over $F$.
(c) Let $G \leq \mathrm{Aut}(K)$ be a finite group and $F = K^G$ be the corresponding fixed field. Then $K/F$ is Galois and $\mathrm{Gal}(K/F) = G$.

3) Equivalence of two definitions of a Galois extension.
4) The main theorem of Galois theory. Examples.
5) Finite fields.
6) Galois groups of polynomials, symmetric functions. Examples of cubic and quartic polynomials.

## Representation theory of finite groups

1) Linear representation $\rho : G \to \mathrm{GL}_F(V)$ as a finitely generated module over the group ring $FG$, where $F$ is a field.
2) Examples, regular representation.
3) Irreducible, indecomposable and completely reducible representation. Maschke's theorem and Schur's Lemma.
4) Basic properties of characters.
5) The orthogonality of characters, central functions. Decomposition of the regular representation.