MAT 534: HOMEWORK 6

DUE THU OCT 10

Problems marked by asterisk (*) are optional.

***1.** Let $a_1, \dots, a_k \in \mathbb{Z}$ be such that $(a_1, \dots, a_k) = 1$ (the gcd). Prove that there is $k \times k$ matrix M with integer coefficients and determinant 1 such that its first row is $a_1, \dots, a_k \in \mathbb{Z}$.

***2.** Using previous problem, prove that every torsion-free finitely generated abelian group (i.e., the group without elements of finite order) is free. (Another proof of this result is in Lang's book, Theorem 8.4 on p. 46).

***3.** Prove that $\langle a, b \,|\, a^2 = 1, b^3 = 1, aba = b^2 \rangle = S_3$.

***4.** Let S be a finite set, $|S| = n$, and $w_1, \dots, w_k$ — words from the alphabet $S \cup \bar{S}$. Prove that there is a group G with generators $x_1, \dots, x_n$ and relations $r_1 = \dots = r_k = 1$, where r_j is obtained from w_j by replacing s_i by x_i and $\bar{s}_i$ by x_i^{-1}, having the following *universality property*: for every group G' on n generators with these relations there is a surjective homomorphism $f : G \to G'$.