MAT 534: HOMEWORK 3
DUE THU SEP 19

1. Let $K, L \leq G$ be such that L normalizes K, i.e., $lKl^{-1} = K$ for all $l \in L$. Prove that $KL \leq G$ and $KL \cong K \rtimes L/K \cap L$.

2. (a) Let p be a prime number. Classify all groups of order p.
(b) Classify all groups of order 6.
(c) Let p and q be different prime numbers. Classify all Abelian groups of order pq.

3. How many ways are there to group numbers $\{1, \ldots, 2n\}$ into pairs? Order of pairs and order inside each pair is not important. For example, for $n = 2$, there are three ways:

$$(12)(34); \quad (13)(24); \quad (14)(23)$$

(*Hint*: first show that one can define a transitive action of S_{2n} on the set of all such pairings.)

4. Prove that alternating group A_n is generated by cycles of lengths 3.

5. (a) Describe all conjugacy classes in S_5. How many elements are in each conjugacy class?
(b) Describe all conjugacy classes in A_5. How many elements are in each conjugacy class?
(c) Prove that A_5 is simple.

6. Let p and q be primes (not necessarily distinct) with $p \leq q$. Prove that if p does not divide $q-1$, then any group G of order pq is Abelian. (*Hint*: Using the class equation, prove that any noncommutative group G of order pq has an element of order q. This element generate the normal cyclic subgroup H of order q. Study the action of G on H by conjugations and compare the resulting automorphisms of H with the possible automorphisms of a cyclic group of order q.)

7. Describe all Sylow 2-subgroups and 3-subgroups of D_{12}.