MAT 534: HOMEWORK 11

DUE THU NOV 21

Problems marked by asterisk (*) are optional.

1. Let R be a ring with $1 \neq 0$ and M be irreducible module. Prove that $D = \mathrm{End}_R(M)$ is a *division ring* (also called a *skew-field*) — a ring with $1 \neq 0$ such that every non-zero element has a multiplicative inverse. Show that M is a vector space over D.
Hint: Use properties of the irreducible modules from the previous assignment.

2. Let

$$A = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ b_1 & a_1 & a_1 & \cdots & a_1 \\ b_1 & b_2 & a_2 & \cdots & a_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_1 & b_2 & b_3 & \cdots & a_n \end{pmatrix}.$$

Compute $\det A$.

3. Let A be $n \times n$ matrix with 2 on the diagonal, -1 immediately below and above the diagonal and zeros elsewhere. Compute $\det A$.

4. Dummit & Foote, exercises 11 and 12 on p. 376.

5. Dummit & Foote, exercise 13 on p. 455.

6. Consider $\mathbb{C}$ as a two-dimensional vector space over $\mathbb{R}$ and prove that $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \simeq \mathbb{C} \times \mathbb{C}$.

***7.** Let A be $m \times m$ matrix and B be $n \times n$ matrix. Prove that

$$\det(A \otimes B) = \det A^n \det B^m.$$