MAT 534: HOMEWORK 6
DUE THU OCT 8

Problems marked by asterisk (*) are optional.

***1.** Let $a_1, \ldots, a_k \in \mathbb{Z}$ be such that $(a_1, \ldots, a_k) = 1$ (the gcd). Prove that there is $k \times k$ matrix M with integer coefficients and determinant 1 such that its first row is $a_1, \ldots, a_k \in \mathbb{Z}$.

***2.** Using previous problem, prove that every torsion-free finitely generated abelian group (i.e., the group without elements of finite order) is free. (Note we skipped this step in class. Another proof of this result is in Lang's book, Theorem 8.4 on p. 46).

***3.** Prove that $\langle a, b \,|\, a^2 = 1, b^3 = 1, aba = b^2 \rangle = S_3$.

***4.** Let S be a finite set, $|S| = n$, and $w_1, \ldots, w_k$ — words from the alphabet $S \cup \bar{S}$. Prove that there is a group G with generators $x_1, \ldots, x_n$ and relations $r_1 = \cdots = r_k = 1$, where r_j is obtained from w_j by replacing s_i by x_i and $\bar{s}_i$ by x_i^{-1}, having the following *universality property*: for every group G' on n generators with these relations there is a surjective homomorphism $f : G \to G'$.