MAT 534: HOMEWORK 3

DUE THU SEP 17

Problems marked by asterisk (*) are optional.

1. (a) Let p be a prime number. Classify all groups of order p.

(b) Classify all groups of order 6.

(c) Let p and q be different prime numbers. Classify all Abelian groups of order pq.

2. How many ways are there to group numbers $\{1, \dots, 2n\}$ into pairs? Order of pairs and order inside each pair is not important. For example, for $n = 2$, there are three ways:

$$(12)(34); \quad (13)(24); \quad (14)(23)$$

(*Hint*: first show that one can define a transitive action of S_{2n} on the set of all such pairings.)

3. Let $\sigma \in S_9$ be defined by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 8 & 5 & 9 & 7 & 4 & 1 & 2 & 6 \end{pmatrix}$$

(a) Find the cycle decomposition of σ. What is the order of σ?

(b) Find the sign of σ.

4. Prove that alternating group A_n is generated by cycles of lengths 3.

5. (a) Describe all conjugacy classes in S_5. How many elements are in each conjugacy class?

(b) Describe all conjugacy classes in A_5. How many elements are in each conjugacy class?

(c) Prove that A_5 is simple.

6. Let p and q be primes (not necessarily distinct) with $p \leq q$. Prove that if p does not divide $q-1$, then any group G of order pq is Abelian. (*Hint*: Using the class equation, prove that any noncommutative group G of order pq has an element of order q. This element generate the normal cyclic subgroup H of order q. Study the action of G on H by conjugations and compare the resulting automorphisms of H with the possible automorphisms of a cyclic group of order q.)

***7.** From Dummit and Foote, problems 5, 8 on p. 130, problems 24, 27 on p. 131 and problems 30, 33 on p. 132.