MAT 312/AMS 251 FALL 2015
REVIEW FOR THE FINAL EXAM

General

The final will be in class (room P-131) on Wednesday, Dec. 9, 5:30pm-8:00pm. Final will be cumulative. It will consist of 8–10 problems. It will be a closed book exam: no books, notes, laptops, tablets, cell phones, etc. The list of covered topics and expected skills is given in reviews for Midterms I, II, which were covering up to Chapter 5. Necessary material in Chapter 6 is covered below.

Chapter 6

§§6.1, 6.2 Understand the similarity between the ring of integers $\mathbb{Z}$ and the *polynomial ring* $R = F[x]$, where F is a field (you may think of F as the field $\mathbb{R}$ of real numbers, the field $\mathbb{C}$ of complex numbers, the finite field $\mathbb{Z}_p$, etc.) Understand the similarity between the divisibility of polynomials $f(x), g(x) \in R = F[x]$ and the divisibility of integers $a, b \in \mathbb{Z}$. Be able to carry out the division algorithm ("long division") for polynomials, giving a quotient and a remainder. Be able to carry out the Euclidean Algorithm to calculate the greatest common divisor $d(x)$ of polynomials $f(x), g(x) \in R$ and to write $d(x)$ as a polynomial linear combination of $f(x)$ and $g(x)$. Know Corollary 6.2.3: a polynomial $f(x)$ has a linear factor $(x - \alpha)$ if and only if $f(\alpha) = 0$. This is very useful in finite fields, since there are only finitely many possible α. Know how to find rational roots of polynomials with integer coefficients.

§6.3 Understand the definition of an *irreducible* polynomial on p. 273; the distinction between irreducible and prime is not important in this context. Understand the proof of Theorem 6.3.4 (every polynomial in R can be written as a product of irreducibles) and the difference from Theorem 1.3.3 (unique factorization for integers): an irreducible factor is only determined up to *a nonzero multiplicative constant*. Note that this constant could be pull out in front of the product if we consider only *monic* irreducible polynomials. Understand Examples 1 and 2 on p. 277 completely. Know Fundamental Theorem of Algebra: every non-constant polynomial with complex coefficients has a root, and Corollaries 6.3.5, 6.3.6.

§6.4 Understand how polynomial congruence classes are defined and have many properties similar to that of congruence classes of integers mod n. Understand multiplication and addition in the set $R_f = R/f$ of congruence classes modulo f. Know the definition of a field and know that the set R_f of polynomial congruence classes $\mod f$ is a field if and only if f is irreducible (one direction is Proposition 6.4.3 in the book; the other is not in the textbook). Be familiar with the examples worked out in class:

- $\mathbb{R}[x]/(x^2+1) \simeq \mathbb{C}$
- $\mathbb{Z}_2[x](x^3+x+1)$

Be able to calculate products and inverses of equivalence classes in these and similar cases.