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CORRECTION

# Correction to: Local index theorem for orbifold Riemann surfaces

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The main result of [1], cf. Introduction and Theorem 2, Formula (3.13), should have the form

$$c_1(\lambda_{-k}, ||\cdot||^{Q}_{-k}) = \frac{6k^2+6k+1}{12\pi^2}\omega_{\mathrm{WP}} - \frac{1}{9}\omega_{\mathrm{cusp}} + \frac{1}{4\pi}\sum_{j=1}^{l}\left(2k\left(B_1\left(\left\{\frac{k}{m_j}\right\}\right)+\frac{1}{2m_j}\right) - m_j\left(B_2\left(\left\{\frac{k}{m_j}\right\}\right)-\frac{1}{6m_j^2}\right)\right)\omega_j^{\mathrm{ell}},$$

where $B_1(x) = x - \frac{1}{2}$ and $B_2(x) = x^2 - x + \frac{1}{6}$ are Bernoulli polynomials, and $\{x\}$ denotes the fractional part of $x$.

Namely, in the proof of Theorem 2 in [1] the term with $B_1(x)$ was inadvertently omitted. This term appears after integration by parts of the integral $I_4$ in formula (3.17). Specifically, let $z_0 \in \mathbb{H}$ be an elliptic fixpoint of order $m$ and for $z \in \mathbb{H}$ let

$$u = \frac{z - z_0}{z - \bar{z}_0}$$

be the variable in the unit disk. Then, as in the proof of Lemma 2 in [1], it is easy to get the following asymptotic as $u \to 0$:

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$$\frac{\partial}{\partial\bar{z}}R_{-k}(z)\left(\frac{dz}{du}\right)^2\overline{\left(\frac{dz}{du}\right)} = -\frac{2km}{\pi}\left(B_1\left(\left\{\frac{k}{m}\right\}\right)+\frac{1}{2m}\right)\frac{1}{u}+O(1).$$

Finally, a simple computation, similar to that for the integral $I_6$ in [1], gives an extra contribution with the first Bernoulli polynomial.

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## Reference

1. Takhtajan, L., Zograf, P.: Local index theorem for orbifold Riemann surfaces. Lett. Math. Phys. **109**(5), 1119–1143 (2019)