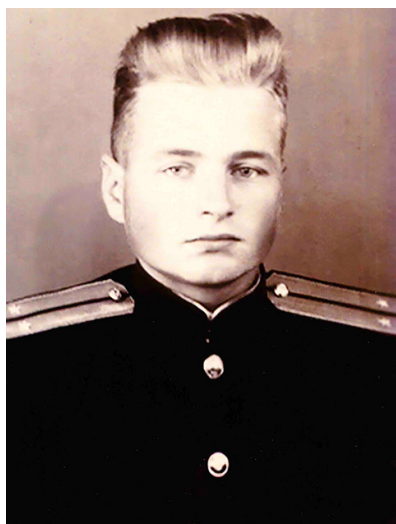


# Askold Ivanovich Vinogradov (1929–2006)

Askold Ivanovich Vinogradov made major contributions to number theory. His most famous result is the celebrated Bombieri–Vinogradov theorem on the distribution of primes in arithmetic progressions, averaged over a range of moduli. In many problems of analytic number theory this result replaces the generalized Riemann hypothesis. Vinogradov proved this theorem in *On the density hypothesis for Dirichlet  $L$ -series*,<sup>1</sup> and independently of him, Enrico Bombieri proved it<sup>2</sup> in *On the large sieve*;<sup>3</sup>

Bombieri's proof is based on the development of Linnik's large sieve method.



Askold Ivanovich Vinogradov was born on October 1, 1929, in the Vsevolozhskiy district of the Leningrad region, in the settlement near Shlisselburg Fortress.<sup>4</sup> His parents, Maria Alexandrovna Sorikhina (Vinogradova) and Ivan Georgievich Vinogradov, were the descendants of peasants from the Tver governance. During WWII, the settlement, where young Askold lived with his mother and younger sister Diana, was near the front line: the ring around the Siege of Leningrad had closed on September 8, 1941, with the right bank of the Neva and the Schlisselburg fortress

controlled by Soviet troops and the left bank by the Germans. In February 1942, Askold ran<sup>5</sup> away from home across Lake Ladoga, was detained in Vologda, and sent to an orphanage. In the summer of the same year, he

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<sup>1</sup> See *Izvestia AS USSR. Mathematical Series*. 29 (1965), P. 903–934.

<sup>2</sup> For this and other works, Enrico Bombieri was awarded the Fields Medal in 1974, see <https://www.mathunion.org/fileadmin/IMU/Prizes/Fields/1974/index.html>

<sup>3</sup> See *Mathematika*. 12 (1965). P. 201–225.

<sup>4</sup> The fortress, also called Oreshek, served as a prison until 1919. Alexander Ulyanov, Lenin's elder brother, was executed there for an assassination attempt on Alexander II. After the revolution, the nearby village on the right side of the Neva was converted into a workers' settlement named after the member of "Narodnaya Volya" N.A. Morozov. There was a gunpowder factory there, where Ivan Georgievich Vinogradov probably worked.

<sup>5</sup> That is according to his 1952 autobiography. However, in his 1991 autobiography, Vinogradov wrote that he was evacuated from Leningrad via the "Road of Life" in March 1942.

was sent to another children's home in the Vologda Region, where he finished 7th grade in 1945. His mother and sister were evacuated from Leningrad in March of 1942. His mother found him in 1944; in June 1945, he left the children's home to join his mother, and in 1947 his mother and Diana returned to Leningrad.

Let us follow with a direct quotation from the 1991 autobiography (that is kept in the LOMI<sup>6</sup> archives):

In 1945, I entered the Baku Naval Preparatory School. Two years later, our school relocated to Kaliningrad (former Königsberg), where I graduated from the Preparatory School in 1948 and was then transferred to the 1st year of the 2nd Baltic Higher Naval School,<sup>7</sup> which was based on and meant as the continuation of our Preparatory School. I graduated from Higher Naval School in 1952 with a torpedoman specialty and received an officer-torpedoman certificate, №743413. I was transferred to the reserve during our allocation and, by special order of the Minister of the Navy, was sent to Moscow, to the postgraduate school of the V.A. Steklov Mathematical Institute of the USSR Academy of Sciences to Academician Ivan Matveyevich Vinogradov. Although the whole thing was done at the special request of the then President of the USSR Academy of Sciences S.I. Vavilov, I.M. Vinogradov and Y.V. Linnik were in fact behind it. These two men followed my fate throughout the years that I studied at the Naval School.

So, after seven years of military experience, on November 15, 1952, young Askold Vinogradov entered postgraduate school at the Steklov Institute of Mathematics with I.M. Vinogradov as his supervisor. Askold learned the famous I.M. Vinogradov method in number theory and, according to his friends, told his senior namesake, "Now I will solve problems using your method."

I.M. Vinogradov said in response,

were there any interesting ones [problems], I would have solved them myself. I will send you to postgraduate school in Leningrad to study other things under Y.V. Linnik's supervision.

On February 15, 1953, Vinogradov was transferred to LOMI, and Linnik became his scientific supervisor.

On December 24, 1953, A.I. Vinogradov passed the postgraduate examination with the grade "excellent," answering three questions in front of a commission consisting of Y.V. Linnik, L.V. Kantorovich, and A.A. Markov. The questions were the following: the Cauchy integral, the Fourier transform, and the notion of the Jordan curve. In the report for 1953, there is a list of books that Vinogradov read during his first year of postgraduate study at LOMI:

<sup>6</sup> LOMI (now POMI or PDMI) is the Russian abbreviation for the Leningrad branch of the V.A. Steklov Mathematical Institute of the USSR Academy of Sciences.

<sup>7</sup> 42nd Baltic Higher Naval School of Surface Navigation (unit 78347)

Fichtenholz' *Differential and Integral Calculus*, in three volumes; Privalov's *Introduction to Complex Analysis*; Vinogradov's *Method of Trigonometric Sums in Number Theory*; Ingham's *The Distribution of Prime Numbers*; and Titchmarsh's *The Riemann Zeta Function*.

When the 1954 annual report of LOMI was being discussed, Linnik characterized Vinogradov in the following way: "I think Askold Ivanovich will do well with his subject: he is an assertive man."

Having defended his Ph.D. thesis, *Additive Problems with Two Prime Numbers and Additional Terms* in 1955, Vinogradov was retained at LOMI as a junior researcher. At that time, Linnik was interested in algebraic geometry and conducted a seminar on André Weil's *Foundations of Algebraic Geometry*.

The thing is, that from the Riemann hypothesis for zeta-functions of curves over finite fields, proved by André Weil in 1948, follows the best possible estimate for special trigonometric sums, the Kloosterman sums. Weil's estimate is inaccessible for the usual methods of the theory of trigonometric sums, including the powerful I.M. Vinogradov's method, so Linnik wanted to understand and generalize Weil's result. Vinogradov recalled that the keynote speaker of the seminar was Linnik's student Boris Skubenko, later a remarkable expert in the geometry of numbers, and each talk began with Linnik's request, "Boris Faddeyevich, remind us, please, the definition of sheaves."<sup>8</sup>

On January 4th, 1963, Askold Ivanovich defended his habilitation<sup>9</sup> thesis, *A Study of Properties of Euler Products for Zeta Functions of Various Algebraic Number Fields and Their Application to Problems of Analytic and Algebraic Number Theory*; from 1964 onwards, he worked as a senior researcher at LOMI (and as a leading researcher from 1986). As mentioned above, in 1965 he proved the density conjecture for the Dirichlet L-series, averaged over the moduli. This remarkable result was not properly appreciated at the Steklov Institute at the time; only a quarter of a century later, A.I. Vinogradov was awarded the I.M. Vinogradov Prize for this work.

From the mid-1960s onwards, Vinogradov's research interests broadened considerably. For instance, in 1967, he tried (unsuccessfully) to prove the well-known Kummer's conjecture on the distribution of the arguments of cubic Gauss sums and proposed an interesting approach to the analytic continuation of the Artin L-function and its connection with the reciprocity laws. The year 1973 was a turning point in his scientific philosophy when he attended L.D. Faddeev's lectures on the Selberg trace formula at a mathematical school in Vilnius. Vinogradov was one of the first to realize that the spectral theory of automorphic functions can provide a powerful new method in analytic number

<sup>8</sup> The story is possibly apocryphal: Ludwig Faddeev said that when he was giving lectures at LSU at the student seminar on quantum field theory, Olga Ladyzhenskaya began each talk with "Ludwig, please remind us the definition of creation and annihilation operators."

<sup>9</sup> The Russian "doctor nauk" degree has no academic equivalent in North America; it is a higher doctoral degree and is roughly comparable to the German *Habilitation*, the French HDR, and British higher doctorates.

theory, and he devoted himself wholly to that subject.<sup>10</sup> A complete list of A.I. Vinogradov's papers can be found on his page at Math-Net.Ru [1].

At this point, a friendship began between Askold Vinogradov, Ludwig Faddeev, and Ludwig's young students, Aleksei Venkov and Leon Takhtajan. Vinogradov participated actively in life at LOMI and organized a weekly seminar on modern number theory with Aleksei Venkov, Nikolai Proskurin, Maxim Skriganov, and Leon Takhtajan. Boris Venkov and Boris Skubenko gave talks at the seminar; Sergei Stepanov and Andrei Tyurin came from Moscow to participate, along with many others. Askold Vinogradov actively supported Nikolai Kuznetsov during the difficult moments of his life. New ideas led N.V. Kuznetsov to the "Kuznetsov trace formula," one of the basic elements of modern analytic number theory. Young Viktor Bykovskii, a graduate of Moscow State University and a student of N.M. Korobov, also came from Moscow. He was at a crossroads, as I.M. Vinogradov had advised him to "improve" the zero-free region of the Riemann zeta-function. Vinogradov introduced Bykovskii to Kuznetsov's trace formula, and from that moment on, Viktor's successful work in number theory has started: in 1982, Ludwig Faddeev submitted Bykovskii's paper to the *Doklady of the USSR Academy of Sciences*; now, V.A. Bykovskii is a well-known Russian mathematician, a corresponding member of the Russian Academy of Sciences.



Vinogradov at work.

The joint paper, *Zeta function of the additive divisor problem and the spectral decomposition of the automorphic Laplacian*, by Vinogradov and

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<sup>10</sup> Starting with the pioneering work by Nikolai Kuznetsov on Kuznetsov's trace formula, the method has been successfully applied to various problems in number theory. We refer the reader to the papers by the Polish-American mathematician Henryk Iwaniec and the South African-born American mathematician Peter Sarnak and their co-authors working in the USA.

Takhtajan, was recognized as the best mathematical work of the year 1984 in the Mathematics Department of the Academy of Sciences of USSR.

A.I. Vinogradov was actively involved in the scientific and public life at LOMI and participated in all the informal activities of the Laboratory of Mathematical Problems in Physics, led by L.D. Faddeev. The geography of the offices on the 5th floor of LOMI facilitated interactions: Ludwig Faddeev in office 506, Aleksei Venkov and Leon Takhtajan in 507, Askold Vinogradov in 508. Askold Ivanovich came on “presence days,”<sup>11</sup> Mondays and Thursdays, and his office became the center of interactions on scientific, literary, and socio-political subjects. Vinogradov traditionally hosted tea parties, of which Boris Venkov and Boris Skubenko were frequent participants, gathering friends both from neighboring offices and from other floors of LOMI.

While Ivan Matveyevich Vinogradov was alive, Askold Ivanovich had good relations with the people at MIAN (the Steklov Mathematical Institute at Moscow), but they took a turn for the worse because of the conflicts at MIAN after I.M. Vinogradov death in 1983. So, in 1987, Vinogradov moved to Khabarovsk to join Kuznetsov and Bykovskii; on February 15, 1987, he became a chief researcher at the Institute for Applied Mathematics of the Far East Branch of the USSR Academy of Sciences. In December of 1991, he returned to LOMI. During these years, Vinogradov worked only on the major problems of number theory, remaining confident in the possibilities of the spectral theory of automorphic functions till the end of his days.

Askold Ivanovich Vinogradov was a unique person: a naval officer and a true gentleman, in the 19th-century sense of the word, a famous mathematician, a pleasant companion for conversation and discussion who never raised his voice, a fan of ballet and rhythmic gymnastics (in his younger years) and a sambo<sup>12</sup> athlete who could deal with any bully. While studying at the Naval School, he was a member of the Komsomol.<sup>13</sup> In 1956, he went to Kazakhstan to help with the harvest on tselina<sup>14</sup> land, and in 1957–1961 he was an instructor of air defense at LOMI. He did not join the Communist Party, as he was critical of the many aspects surrounding its policies and ideology. At the same time, he was a patriot of his country and had a deep respect for the Great Patriotic War and the Supreme Commander of the Soviet Army. Like Boris Faddeyevich Skubenko, he was very keen on war memoirs and avidly read the memoirs of both Soviet commanders and German generals.

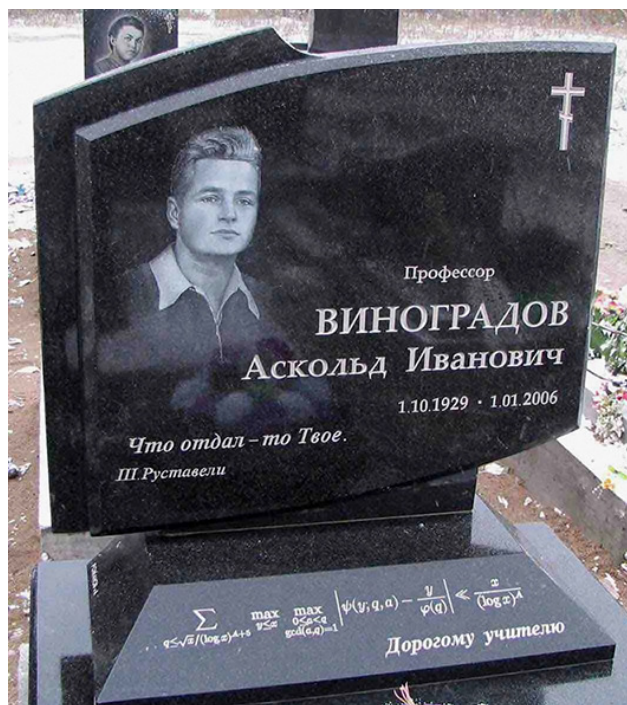
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<sup>11</sup> In academic institutions, there was usually no need to come to work every day: one or two days a week were compulsory and called “presence days,” and on the other days, called “library days,” academics could go to the library or work at home.

<sup>12</sup> Sambo is a martial art that originated in the Soviet Union in the 1920s.

<sup>13</sup> Komsomol was a political youth organization in the Soviet Union, de facto the youth division of the Communist Party, although officially independent.

<sup>14</sup> Tselina or virgin lands is an umbrella term for underdeveloped, sparsely populated, highly fertile lands often covered with chernozem soil, which were mostly located in the steppes of the Volga region, Northern Kazakhstan, and Southern Siberia.



Vinogradov's tomb.

His lifestyle was very modest, and he was consistent in all his habits. For example, during Soviet times he used to go to Novy Svet in Crimea every summer, tried to walk as much as possible, regularly came to Komarovo, and during his walks with Takhtajan liked to discuss new approaches to various problems in number theory. As for drinks, he could allow himself a little champagne, being at the same time indulgent towards the tastes and predilections of his colleagues. He was very generous and always helped old friends who were in trouble. The Khabarovsk period of Vinogradov's life is reflected upon in a review by V.A. Bykovskii [2].

Askold Ivanovich would often marvel at descriptions in the tabloids of "contact with aliens" and people allegedly "abducted for experiments." He used to say, "If I had met them, I would immediately ask how to prove the Riemann hypothesis!" Perhaps now he knows...

*Leon Takhtajan*

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