

MAT 126 Calculus B Fall 2005 Practice Midterm I

Name: _____

I.D.: _____ Section number: _____

Answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. The actual exam will contain 5 problems. This practice test contains more problems to give you more practice.

1. (a) (10 points) Estimate the area under the graph of $f(x) = 16 - x^2$ from $x = 0$ to $x = 4$ using four rectangles and right endpoints. Sketch the graph and rectangles. Is your estimate and underestimate or an overestimate?
- (b) (10 points) Repeat part (a) using left endpoints.
2. (a) (10 points) Evaluate integral by interpreting it as area

$$\int_{-5}^5 \sqrt{25 - x^2} dx$$

- (b) (5 points) Determine a region whose area is equal to

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\pi}{4n} \tan \frac{i\pi}{4n}$$

Do not evaluate the limit.

3. Given two functions $f(x)$ and $g(x)$ which satisfy

$$\begin{aligned} \int_0^3 f(x) dx &= 5, & \int_0^5 f(x) dx &= 7, \\ \int_3^5 g(x) dx &= 1, & \int_0^5 g(x) dx &= 9, \end{aligned}$$

find

- (a) (5 points)

$$\int_3^5 (3f(x) - g(x)) dx$$

- (b) (5 points)

$$\int_0^3 (f(x) + 2g(x)) dx$$

4. (5 points) Express the limit as a definite integral on the given interval $[0, 4]$:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{e^{x_i}}{1 + x_i} \Delta x$$

5. Evaluate the following indefinite integrals
(a) (5 points)

$$\int (3 \cos x - 4 \sin x) dx$$

- (b) (10 points)

$$\int \frac{\cos x}{1 - \cos^2 x} dx$$

6. Evaluate the following definite integrals
(a) (5 points)

$$\int_1^2 x^{-2} dx$$

- (b)

$$\int_1^8 \frac{x-1}{\sqrt[3]{x^2}} dx$$

- (c) (5 points)

$$\int_1^{27} \frac{1}{9t} dt$$

- (d) (5 points)

$$\int_{\ln 3}^{\ln 6} 5e^x dx$$

- (e) (10 points)

$$\int_{\pi/3}^{\pi/2} \csc x \cot x dx$$

7. Let

$$F(x) = \int_{x^3}^8 e^{-t^2} dt + \sqrt{x}$$

- (a) (10 points) Find $F'(x)$
(b) (5 points) Find $F(2)$

8. (10 points) Find a function $f(x)$ and a number a such that

$$3 + \int_a^x \frac{f(t)}{t} dt = \sqrt[3]{x}$$

for all $x > 0$.