Practice Final Exam
MAT 125 Spring 2008

Answer each question in the space provided and on the back of the sheets. Write full **solutions**, not just answers: unless otherwise marked, **answers without justification will get little or no partial credit**. Cross out anything the grader should ignore and circle or box the final answer. Do **NOT** round answers.

No books, notes, or calculators!

1. Compute the following limits by distinguishing between "$\lim f(x) = \infty$", "$\lim f(x) = -\infty$", and "limit does not exist even allowing for infinite values".

   (a) $\displaystyle\lim_{x\to -1} x^3 + 7x^2 - 1$

   (b) $\displaystyle\lim_{x\to\infty} x \tan\frac{1}{x}$

   (c) $\displaystyle\lim_{x\to 5} \frac{x^2 - 4x - 5}{x - 5}$

   (d) $\displaystyle\lim_{x\to 0} x^4 \cos\frac{\pi}{x}$

   (e) $\displaystyle\lim_{x\to 0} \frac{e^x - 1 - x - x^2/2}{x^3}$

   (f) $\displaystyle\lim_{x\to\infty} \frac{x^3 + 1001x + 77}{x^3 - x^2 + 99}$

   (g) $\displaystyle\lim_{x\to\pi/2} \frac{\cos x}{2x - \pi}$

   (h) $\displaystyle\lim_{x\to\infty} (xe^{1/x} - x)$

2. Compute the derivatives of the following functions

   (a) $f(x) = x^3 - 12x^2 + x + 137\pi$

   (b) $f(x) = (2x + 1)\sin x$

   (c) $g(s) = \sqrt{1 + e^{2s}}$

   (d) $h(t) = \dfrac{1 + e^t}{1 - e^t}$

   (e) $f(x) = (2x + 2)^{100}$

   (f) $g(x) = x^{\sin x}$

3. Let $f(x) = xe^{-x^2}$.

   (a) Find asymptotes of $f(x)$ (hint: $f(x) = \dfrac{x}{e^{x^2}}$)

(b) Compute the derivative of $f(x)$

(c) On which intervals is $f(x)$ increasing? decreasing?

(d) Sketch a graph of $f(x)$ using the results of the previous parts and the fact that $f(0) = 0$.

4. Let $f(x) = -2x^3 + 6x^2 - 3$.

   (a) Compute f', f''.

   (b) On which intervals is $f(x)$ increasing/decreasing?

   (c) On which intervals is $f(x)$ concave up/down?

   (d) Find all critical points of $f(x)$. Which of them are local maximums? local minimums? neither? Justify your answer.

5. It is known that the polynomial $f(x) = x^3 - x - 1$ has a unique real root. Between which two whole numbers does this root lie? Justify your answer.

6. It is known that for a rectangular beam of fixed length, its strength is proportional to $w \cdot h^2$, where w is the width and h is the height of the beam's cross-section. Find the dimensions of the strongest beam that can be cut from a $12''$ diameter log (thus, the cross-section must be a rectangle with diagonal $12''$).

7. The curve defined by the equation

$$y^2(y^2 - 4) = x^2(x^2 - 5)$$

   is known as the "devil's curve". Use implicit differentiation to find the equation of the tangent line to the curve at the point $(0; -2)$.

8. Find the most general function $f(x)$ satisfying

   (a) $f''(x) = \cos x$

   (b) $f'(x) = \dfrac{x^2 + x + 1}{x}$