8.5

6.

$$\sum_{n=1}^{\infty}\frac{x^n}{n^2}\ ,\ Let\quad a_n=\frac{x^n}{n^2}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{|x^{n+1}|}{(n+1)^2}\cdot\frac{n^2}{|x^n|}=\lim_{n\to\infty}\frac{n^2}{(n+1)^2}|x|$$

$$=|x|$$

If |x|<1, then the series is convergent. Therefore, the radius of convergence is 1.

8.

$$\sum_{n=1}^{\infty}\frac{x^n}{n\cdot 3^n}\ ,\ Let\quad a_n=\frac{x^n}{n\cdot 3^n}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{|x^{n+1}|}{(n+1)\cdot 3^{n+1}}\cdot\frac{n\cdot 3^n}{|x^n|}=\lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{|x|}{3}$$

$$=\frac{|x|}{3}$$

If $\frac{|x|}{3}<1$ then the series is convergent. Therefore, the radius of convergence is 3.

18.

$$\sum_{n=1}^{\infty}\frac{nx^n}{1\cdot 3\cdot 5\cdots\cdots(2n-1)}\ ,\quad Let\ a_n=\frac{nx^n}{1\cdot 3\cdot 5\cdots\cdots(2n-1)}$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{(n+1)|x^{n+1}|}{1\cdot 3\cdot 5\cdots\cdots(2n-1)(2n+1)}\cdot\frac{1\cdot 3\cdot 5\cdots\cdots(2n-1)}{n|x^n|}$$

$$=\lim_{n\to\infty}\frac{n+1}{(2n+1)n}|x|$$

$$=0$$

Hence, for any x the series is convergent. Therefore, the radius of convergence is infinite.

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Let R be the radius of convergence of the series, $\sum_{n=0}^{\infty} c_n x^n$. Then $4 \le R < 6$

$a) \sum_{n=0}^{\infty} c_n$ $\qquad x = 1$

$b) \sum_{n=0}^{\infty} c_n 8^n$ $\qquad x = 8$

$c) \sum_{n=0}^{\infty} c_n (-3)^n$ $\qquad x = -3$

$d) \sum_{n=0}^{\infty} (-1)^n c_n 9^n = \sum_{n=0}^{\infty} c_n (-9)^n$ $\qquad x = -9$

The radius of convergence R is between 4 and 6. It means that if

$x \in (-4,\ 4)\ or\ |x| < 4$, then the given series is convergent, and if

$x \notin [-6,\ 6]\ or\ |x| > 6$, then the given series is divergent.

Hence, the series a) and c) are convergent. The series b) and d) are divergent.