8.2

10.

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

a) $\sum_{j=1}^{n} a_j = a_1 + a_2 + a_3 + \cdots + a_n$

Therefore, the two given series are exactly same.

b)

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

$$\sum_{i=1}^{n} a_j = a_j + a_j + a_j + \cdots a_j = n \cdot a_j \ \textit{because the subscipt of } \Sigma, i \textit{ is not different from that of } a.$$

12.

$$1 + 0.4 + 0.16 + 0.064 + \cdots$$

Let $a_n = (0.4)^{n-1}$

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (0.4)^{n-1} = \frac{1}{1-0.4} = \frac{5}{3}$$

Therefoer, the given series is convergent.

20.

$$\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$$

$$\lim_{n\to\infty} \frac{(n+1)^2}{n(n+2)} = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n} = \lim_{n\to\infty} \frac{1 + \frac{1}{n} + \frac{1}{n^2}}{1 + \frac{1}{n}} = 1 \neq 0$$

Therefore, the given series is divergent.

28.

$$\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$$

$$Let\ \ S_n = \sum_{k=1}^{n} \ln \frac{k}{k+1},\ \ then$$

$$S_n = \sum_{k=1}^{n} (\ln k - \ln(k+1)) = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + (\ln 3 - \ln 4) + \cdots + (\ln n - \ln(n+1))$$

$$= \ln 1 - \ln(n+1)$$

$$= -\ln(n+1)$$

$$Thus,\ \ \sum_{n=1}^{\infty} \ln \frac{n}{n+1} = \lim_{n\to\infty} S_n = \lim_{n\to\infty}(-\ln(n+1)) = -\infty$$

Therefore, the given series is divergent.

34.

$$\sum_{n=0}^{\infty} 2^n (x+1)^n = \sum_{n=0}^{\infty} [2(x+1)]^n$$

Thus, the given series is a geometric series. So that the series is convergent if and only if

$$|\,2(x+1)\,| < 1 \ \ \Leftrightarrow\ \ -\frac{3}{2} < x < -\frac{1}{2}$$

(If the given series is divergent, then $\sum_{n=0}^{\infty} 2^n (x+1)^n = \frac{1}{1-2(x+1)}$.)

48.

$\sum_{n=1}^{\infty} a_n$ is convergent. Then $\lim_{n\to\infty} a_n = 0$

However, this condition implies that $\lim_{n\to\infty} \frac{1}{a_n}$ is not convergent.

So that $\lim_{n\to\infty} \frac{1}{a_n} \neq 0.$ Therefore, $\sum_{n=1}^{\infty} \frac{1}{a_n}$ must be divergent.

50.

$\sum a_n$ *and* $\sum b_n$ *are divergent.*

counterexample)

Let $a_n = 1,\ \ b_n = -1,$ *then* $\sum a_n = \infty$ *and* $\sum b_n = -\infty.$

So both $\sum a_n$ *and* $\sum b_n$ *are divergent. However,* $\sum (a_n + b_n) = \sum 0 = 0$

So that $\sum (a_n + b_n)$ *is convergent.*

If you take $a_n = n,$ *and* $b_n = -n,$ *then they are another counterexample.*