8.1

8.

{ 0, 2, 0, 2, 0, 2...}

$a_n = 1 + (-1)^n \ or \ 1 - (-1)^{n+1}$

10.

$a_n = \frac{n+1}{3n-1}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n+1}{3n+1} = \lim_{n\to\infty} \frac{1 + 1/n}{3 + 1/n} = \frac{1}{3}$

14

$a_n = \frac{n}{1+\sqrt{n}}$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{1+\sqrt{n}} = \lim_{n\to\infty} \frac{\sqrt{n}}{1/\sqrt{n} + 1} = \infty$

18.

$a_n = \frac{n \cos n}{n^2+1}$

$|a_n| = \frac{n|\cos n|}{n^2+1} \le \frac{n}{n^2+1}$, *for all* $n > 0$ *because* $|co|$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n|\cos n|}{n^2+1} \le \lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{1/n}{1 + 1/n^2} = 0$

$\lim_{n\to\infty} |a_n| \le 0$ *so that* $\lim_{n\to\infty} |a_n| = 0$

$\lim_{n\to\infty} |a_n|$ *is convergent, then it implies that* $\lim_{n\to\infty} a_n$ *is also convergent.*

Moreover, $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} a_n$

Therefore, $\lim_{n\to\infty} a_n = 0$

22.

Let $a_n = \dfrac{\ln(2+e^n)}{3n}$, *Use the squeeze theorem.*

$e^n < 2+e^n < 2e^n$ *for all* $n>1$

Thus $\lim\limits_{n\to\infty}\dfrac{\ln e^n}{3n} \le \lim\limits_{n\to\infty} a_n \le \lim\limits_{n\to\infty}\dfrac{\ln 2e^n}{3n}$

Firstly, $\lim\limits_{n\to\infty}\dfrac{\ln e^n}{3n} = \lim\limits_{n\to\infty}\dfrac{n}{3n} = \dfrac{1}{3}$

Secondly, $\lim\limits_{n\to\infty}\dfrac{\ln 2e^n}{3n} = \lim\limits_{n\to\infty}\dfrac{\ln 2+\ln e^n}{3n} = \lim\limits_{n\to\infty}\dfrac{\ln 2+n}{3n} = \lim\limits_{n\to\infty}\dfrac{\frac{\ln 2}{n}+1}{3} = \dfrac{1}{3}$

Hence, $\dfrac{1}{3} \le \lim\limits_{n\to\infty} a_n \le \dfrac{1}{3}$

Therefore, $\lim\limits_{n\to\infty} a_n = \dfrac{1}{3}$

26.

$a_n = \dfrac{(-3)^n}{n!}$ *then*

$|a_n| = \dfrac{3^n}{n!}$, $|a_{n+1}| = \dfrac{3^{n+1}}{(n+1)!} = \dfrac{3^n}{n!}\cdot\dfrac{3}{n+1} = |a_{n+1}| = |a_n|\cdot\dfrac{3}{n+1}$

Thus, $|a_n| > |a_{n+1}|$, *for all* $n \ge 3$

So that $|a_n| \le \dfrac{9}{2} = \max\{a_1,\ a_2,\ a_3\}$

Hence, $|a_{n+1}| \le \dfrac{9}{2}\cdot\dfrac{3}{n+1}$

$\lim\limits_{n\to\infty}|a_{n+1}| \le \lim\limits_{n\to\infty}\dfrac{9}{2}\cdot\dfrac{3}{n+1} = \lim\limits_{n\to\infty}\dfrac{27}{2(n+1)} = 0$

So that $\lim\limits_{n\to\infty}|a_n| = 0$

Therefore, $\lim\limits_{n\to\infty} a_n = 0$ *by Squeeze theorem.*

38.

$\{\sqrt{2},\ \sqrt{2\sqrt{2}},\ \sqrt{2\sqrt{2\sqrt{2}}},\ \cdots\}$

$\sqrt{2} = 2^{\frac{1}{2}}$

$\sqrt{2\sqrt{2}} = 2^{\frac{1}{2}+\frac{1}{4}}$

$\sqrt{2\sqrt{2\sqrt{2}}} = 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}}$

The limit of the given sequence is the same as following.

$2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots}$

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$$\textit{Therefore,}\ \lim_{n\to\infty} 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}+\cdots} = 2^1 = 2$$

40.

$$a_n = \frac{2n-3}{3n+4}$$

$$a_{n+1} - a_n = \frac{2(n+1)-3}{3(n+1)+4} - \frac{2n-3}{3n+4} = \frac{2n-1}{3n+7} - \frac{2n-3}{3n+4} = \frac{(2n-1)(3n+4)-(2n-3)(3n+7)}{(3n+7)(3n+4)}$$

$$= \frac{17}{(3n+7)(3n+4)} > 0 \qquad \textit{for all}\quad n \geq 1$$

So that $a_{n+1} > a_n$, Hence, a_n is an increasing sequence.

$a_1 = -\frac{1}{7}$, *and* $a_n > 0$ *for all* $n \geq 2$.

Moreover, $a_n = \frac{2n-3}{3n+4} < \frac{2n+2}{3n+3} = \frac{2}{3}$. *Thus* $-\frac{1}{7} \leq a_n < \frac{2}{3}$

So that a_n *is bounded*.