Solution 7.6

2.

In these problems, the sign of each term are very crucial. In particular, the sign of xy term determine the property of the differential equations.

a)

$$\frac{dx}{dt} = 0.12x - 0.0006x^2 + 0.00001xy$$
$$\frac{dy}{dt} = 0.08x + 0.00004xy$$

, that is,

$$\frac{dx}{dt} = 0.12x\left(1 - \frac{1}{200}x + \frac{1}{12000}y\right)$$
$$\frac{dy}{dt} = 0.08x\left(1 + \frac{1}{2000}y\right)$$

-------- (*)

In this equation, dy/dt is positively proportional to the number of x and y. Moreover, dy/dt is much more dependent of the number of x than y, so that 0.00004xy is ignorable (it is necessary that the sign of xy term is positive.)

Thus the rate of increase of y is almost positively dependent of the number of x.

Without the term of xy, dx/dt is the logistic equation and the equilibrium number of x is 200. It means that whichever number of the initial value, it converge the 200.
Furthermore, In the equation (*),

$$if \ \frac{dx}{dt} = 0 \ and \ \ x \neq 0, \ \ then \ \ 1 - \frac{1}{200}x + \frac{1}{12000}y = 0 \ \ or \ \ 12000 - 60x + y = 0$$

Hence, in the phase space, for any given point (x, y), it goes to the line $12000 - 60x + y = 0$ and along this line both x and y increase when the t increase.

Near that line y begins to increase rapidly because $x \approx 200$ then $1 - \frac{1}{200}x \approx 0$ and it implies that dx/dt almost proportional to xy. The beginning point of the y which is rapidly increase is dependent of the ratio of constants of $x^2 \ term \ \ and \ \ xy \ term$.

The whole shape of integral curves in phase space is similar to the upside-down shape of 2 dimensional fountains whose support is the line $12000 - 60x + y = 0$.

Therefore, these equations are a cooperation model.

If you see the phase space of above differential equation, visit the web page
www.math.psu.edu/melvin/phase/newphase.html

b)

$$\frac{dx}{dy} = 0.15x - 0.0002x^2 - 0.0006xy$$

$$\frac{dy}{dt} = 0.2y - 0.00008y^2 - 0.0002xy$$

$$\left.\begin{aligned} \frac{dx}{dt} &= 0.15x\left(1 - \frac{1}{750}x - \frac{1}{250}y\right) \\ \frac{dy}{dt} &= 0.2y\left(1 - \frac{1}{2500}y - \frac{1}{1000}x\right) \end{aligned}\right\} \text{-----------(*)}$$

In this equation, if $dx/dt = 0$ *and* $dy/dt = 0$,

then $(x, y) = (0,0),\ (750,\ 0)\ or\ (0,2500)$ because x and y are non-negative numbers.

Without xy term, both dx/dt and dy/dt are logistic equations and equilibrium point (x, y) = (750, 2500).

In this equation (*),

if $y > 250$ then $dx/dt < 0$, that is, the number of x decreases. Moreover, y can increase 2500, so that eventually, the number of x decrease and it will be extinguished.

If $x > 1000$ then $dy/dt < 0$, that is, the number of y decreases. However, x goes to 750, so that eventually the number of y will converge to 2500.

Thus in the phase space, all values converge to (0,2500).

When the value of x is very small, y begin to converge to 2500 rapidly.

Hence, x will be eliminated and y will be survived.

Each number of x and y makes negative effect to each other.

Therefore, these equations are a competitive model.

If you see the phase space of above differential equation, visit the web page
www.math.psu.edu/melvin/phase/newphase.html

If you want to see the general answers and solutions of differential equations, for example, predator-prey model and competitive model, then you can refer the following book.

Elementary differential equations and boundary value problems

Written by Boyce and DiPrima (This book is in Math Learning Center.)