REVIEW FOR MIDTERM II; MAT 312 (SPRING, 08)

(1) Let X denote an infinite set. Suppose that there is a surjective map $f : \mathbb{P} \longrightarrow X$ from the positive integers $\mathbb{P}$. Show that X is a "countable set".

(2) Construct a surjective map $f : \mathbb{R} \longrightarrow \mathbb{P}$ from the real numbers $\mathbb{R}$.

(3) Let $X, <$ denote a strict partially ordered set.

(a) If X is a finite set show that there always exits at least on minimal element and at least one maximal element.
(b) Give an example (of a finite strict partially ordered set) where there are more than one minimal elements and more than one maximal elements.
(c) Give an example (of an infinite strict partially ordered set) where there is not maximal elements and no minimal elements.

(4) Set $X = \{1, 2, 3, ..., 65, 66, 67\}$ and set $Y = \{A \subset X \mid \{1, 2, 3, 4\} \subset A\}$. Express $\mid Y \mid$ as a power of 2.

(5) Set $X = \{2, 3, 4, 5, 6, 7, ...\}$ and define a relation R on X by
$xRy \quad \Leftrightarrow \quad x \mid y$ and $x \neq y$.

(a) Show that R is a strict partial ordering on X.
(b) Which elements of X are the minimial elements? For each minimal element describe its immediate successors.

(6) Let $f : X \rightarrow Y$ denote a function and define a relation R on X by $xRy \quad \Leftrightarrow \quad f(x) = f(y)$. Show that R is an equivalence relation.

(7) Let $Perm(X)$ denote the set of permutations of the set X, and let $\sigma, \tau \in Perm(X)$ denote two disjoint permutations.

(a) Show that for any positive integer k the σ^k, τ^k are also disjoint permutations.
(b) Show that if $\sigma \circ \tau = id$ then both $\sigma = id$ and $\tau = id$ hold.

(8) Define $\sigma \in S(7)$ by

$$\sigma = (2, 3)(7, 5, 3, 4, 2, 6)(1, 2, 3, 4, 5)$$

and define $\tau \in S(7)$ by the matrix

$$1, 2, 3, 4, 5, 6, 7$$

$$7, 1, 2, 6, 3, 5, 4$$

(a) Compute σ^{-3}.
(b) Compute $\sigma\tau\sigma^2$.

(c) Write both σ and τ as a product of disjoint cycles.
(d) Compute the orders of σ and τ.
(e) Compute $sgn(\sigma)$ and $sgn(\tau)$.
(f) Compute τ^{88}.