Homework 10 : §5.2 - $1^*, 2, 3, 4^*, 5^*$.
$*$ - in the back of the book

Exercises 5.2

2. It follows from the definition of Euler's phi function that if $m|n$ then $\phi(m)|\phi(n)$. Let $n \geq 3$ and choose the highest prime factor of n, say p. Then either p is odd, in which case $2|\phi(p)$ and hence $2|\phi(n)$ or $p = 2$ and $n = 2^r$ for some $r \geq 2$. In this case, $\phi(n) = 2^{r-1}$ is divisible by 2.

3. Fix a square and label the vertices in a clockwose order as $1, 2, 3, 4$. Let x denote the clockwise rotation by $2\pi/4$ and y denote the reflection across the line joining 1 and 3. The group generated by x and y is $D(4)$. It is of order 8 and has relations $x^4 = 1 = y^2, xyx^{-1} = y$. For any reflection τ, let $H = \{1, \tau\}$ denote the subgroup it generates. Then its cosets are H, xH, x^2H and x^3H.