MAT 310: HW4

(1) Let $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ denote rotation about the origin of the plane thru an angle of θ radians. Using the geometric definition of vector addition and scalar multiplication in the plane (see pages 2,3), give a geometric proof that T is a linear transformation. (Your proof might consist of a few pictures.)

(2) In section 2.2 do #2(a)(e)(g),4,8,9,16.

(3) Let $\beta = \{\mathbf{e}_1, \mathbf{e}_2\}$ denote the standard ordered basis for $\mathbb{R}^2$ and set $\gamma = \{\mathbf{e}_2, -\mathbf{e}_1\}$. Compute the matrix $[T]_\gamma^\beta$, where $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ comes from **(1)** above.

(4) In section 2.3 do #2(b),3,11,12,16

(5) Let $T : V \longrightarrow W$ denote a linear transformation between two finite dimensional vector spaces over the real numbers. Prove that there are basis β and γ for V and W respectively such that the matrix $[T]_\beta^\gamma$ satisfies the following properties: every off diagonal element is equal to 0; every diagonal element is equal to either 1 or 0. (In this problem the matrix $[T]_\beta^\gamma$ need not be a square matrix; thus we do not call it a "diagonal matrix" — see page 18 in the text book. Note that this problem is similar to problem #16 on page 86 in the text book.)