REVIEW FOR FINAL EXAM (MAT 132, SPRING 2010)

(1) Find the general solution to the differential equation $\frac{dy}{dx} = \frac{xy^4+y^4}{x^2+2x-8}$.

(2) Consider the differential equation $y' = y + x$.

(a) Show that $y = 1 + x + x^2 + \sum_{n=3}^{\infty} \frac{2x^n}{n!}$ is a solution to the given differential equation.
(b) Find the radius of convergence for the power series in part (a)?
(c) What elementary function does the power series of part (a) converge to?

(3) Find the orthonal trajectories of the family of curves $y^2 = kx^5$, where k is an arbitrary constant.

(4) Which of the following sequences $a_1, a_2, a_3, ...$ converges, and to what values do the convergent sequences converge?

(a) $a_n = (-1)^n 3^{\frac{1}{n}}$
(b) $a_n = \frac{n^3+n^2-1}{8n^3-n+2}$
(c) $a_n = \frac{(ln(n))^2}{n}$

(5) Which of the following series converge?

(a) $\sum_{n=2}^{\infty} \frac{1}{nln(n)}$
(b) $\sum_{n=1}^{\infty} (-1)^n 3^{1/n^2}$
(c) $\sum_{n=1}^{\infty} \frac{n+1}{n^2-n+3}$
(d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$
(e) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$

(6) problem (11) on page 548.

(7) problem (12) on page 548.

(8) problem (14) on page 549.

(9) Let R denote the region in the plane between the graph of $y = 3 + sin(x^2)$ and the x-axis, from $x = 0$ to $x = 2$.

(a) Find an infinite series which converges to the area of R.
(b) Let S denote the 3-dimensional solid obtained by rotating R about the line $x = -1$ in 3-space. Find an infinite series which converges to the volume of S.

(10) Find the interval of convergence for each of the following power series.

(a) $\sum_{n=2}^{\infty} \frac{(x-4)^n}{n^{\frac{2}{3}}}$

(b) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(c) $\sum_{n=3}^{\infty} \frac{(x+1)^n}{3^n}$

(11) Verify that each of the following infinite series converges. For each of the series determine how large n must be in order that the n'th partial sum s_n and the infinite sum s are equal to 3 decimal places.

(a) $\sum_{n=1}^{\infty} \frac{3}{(n+2)^3}$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3}{n+1}$

(12) Compute the Taylor series for each of the following functions at a. Find the radius of convergence for each Taylor series.

(a) $(1 + x)^{\frac{1}{3}}$ at $a = 0$.
(b) $(x - 3)^3 + x^2 - 2$ at $a = -2$.
(c) $ln(x + 3)$ at $a = -1$.
(d) e^{2x^5} at $a = 0$.
(e) $\frac{2x-1}{x^2-2x-8}$ at $a = 0$. (Hint: first use partial fractions to find another expression for this function.)