MAT 127 PRACTICE FINAL

(1) Consider the initial value problem

$$y'' - y' + 3y = 0$$
$$y(0) = 1, y'(0) = -1 \quad .$$

Assuming the solution to this initial value problem has is the power series

$$y = \sum_{n=0}^{\infty} c_n x^n \quad ,$$

find all the coeffiecients c_n for $n \leq 6$.

(2) Use the seperation of variables technique to solve the initial value problem

$$y' = yln(x)$$
$$y(1) = 2 \quad .$$

(3)

(a) Use Euler's Method with step size 1 to estimate the value $y(3)$, where y denotes the solution to the initial value problem

$$y' = y + x^2$$
$$y(0) = 1 \quad .$$

(b) Sketch the direction field for the differential equation given in part (a).

(4) Determine whether or not each of the following sequences $\{a_n\}$ converges. If the sequence converges, then compute the limit.

(a) $a_n = 2 + (-2/\pi)^n$
(b) $a_n = (n^3 - n + 2)/(n^2 - 3n^3)$
(c) $a_n = 3^n/n^4$
(d) $a_n = n^2/n!$

(5) Use any method to determine whether or not each of the following series $\sum_{n=1}^{\infty} a_n$ converges.

(a) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (1 + n^{-2})/n$
(b) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (n^2 + n + 2)/(n - 8n^2)$

(c) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1}(2 + cos(n))/n^2$
(d) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n}(2 + e^{-n})/n$
(e) $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} n^3/2^n$

(6) The differential equation

$$P' = 0.2P(1 - P/1000)$$

describes the change in population of wild Dachshunds over time.

(a) Find the equilibrium solutions for this differential equation.
(b) Sketch the direction field for this differential equation; be sure to indicate the equilibrium solutions in your sketch.

(7) Consider the function $f(x) = 3x^{-2} + 2x - 1$.

(a) Compute the Taylor series for this function at the number 1.
(b) Find the radius of convergence for the Taylor series in part (a).
(c) Find the interval of convergence for the Taylor series in part (a).

(8) A bacteria culture grows with constant relative growth rate. At the outset there are 500 bacteria.

(a) If $y(t)$ denotes the number of bacteria present after t-hours, write down an initial value problem which y satisifies.
(b) If after 3 hours there are 2400 bacteria, then how many bacteria are there after 2 hours?